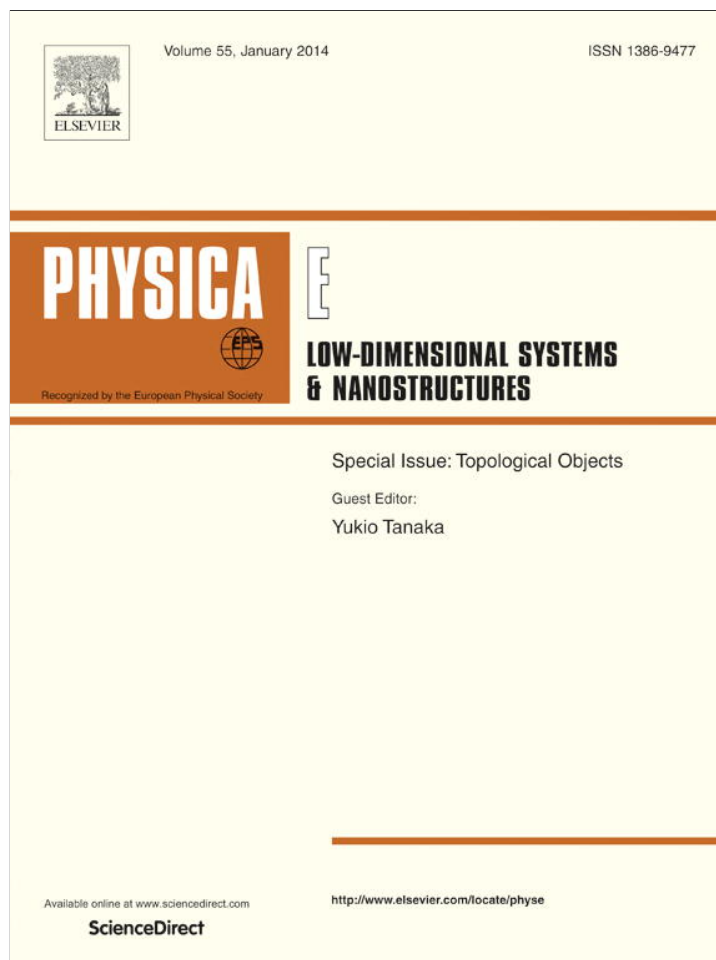


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Bulk-boundary correspondence in Josephson junctions

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ARTICLE INFO

Article history:

Received 19 October 2012

Received in revised form

27 February 2013

Accepted 19 March 2013

Available online 30 March 2013

Keywords:

Topological superconductor

Josephson junction

Bulk-boundary correspondence

ABSTRACT

We discuss Andreev bound states appearing at the interface between two different superconductors characterized by different nontrivial topological numbers such as one-dimensional winding numbers and Chern numbers. The one-dimensional winding number characterizes d_{xy} and p_x wave superconductors. The Chern number characterizes chiral superconductors. The number of interfacial bound states at the zero-energy is equal to the difference between the topological numbers on either sides of the Josephson junction. We also discuss relation between properties of the Andreev bound states at the zero-energy and features of Josephson current at low temperature.

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1. Introduction

According to the topological classification of matter [1], a number of unconventional superconductors have been categorized in terms of nontrivial topological numbers [2,3] such as Z_2 number, Chern number, and one-dimensional winding number. The non-centrosymmetric superconductor [4,5] is topologically nontrivial when the amplitude of spin-triplet helical- p wave order parameter is larger than that of spin-singlet s wave one [6]. Such superconducting phase is characterized by a topological number $Z_2 = 1$. The transport properties of non-centrosymmetric superconductors are qualitatively different depending on Z_2 number [7]. The spin-triplet chiral- p wave superconductivity in Sr_2RuO_4 [8,9] is characterized by Chern numbers $n = \pm 1$ [10]. The Chern numbers here are referred to as Thouless-Kohmoto-Nightingale-den Nijs (TKNN) number in solid state physics [11]. The spin-singlet chiral- d wave ($n = \pm 2$) superconductivity has been suggested in $\text{Na}_x\text{CoO}_2 \cdot y\text{H}_2\text{O}$ [12–15], heavy fermionic compounds [16,17], graphene [18], high- T_c superconductors [19,20], and β -MnCl [21]. Unconventional d_{xy} wave symmetry in high- T_c superconductors and p_x wave symmetry in the polar state in ^3He are characterized by the one-dimensional winding number [22,23] which we call Sato number in this paper.

The unconventional superconductors have subgap Andreev bound states (ABSs) at their surface [24–28], which has been known for some time. Such surface state is responsible for unusual low energy transport in high- T_c superconductors [29–35], chiral- p wave

superconductor [36–41]. In particular in spin-triplet superconductors, the surface states attract much attention these days because they are recognized as Majorana fermion bound states [42–47]. The proximity effect of spin-triplet superconductors is known to be anomalous because of the penetration of the Majorana bound state into a normal metal [48,49].

Today the presence of such surface bound state is explained in terms of the bulk-boundary correspondence of topological superconductivity. According to the bulk-boundary correspondence, the number of the surface bound state at the zero-energy would be identical to the absolute value of topological number defined in the bulk superconductor. In fact, this prediction has been confirmed in a number of theoretical studies. The validity of the bulk-boundary correspondence should be confirmed also in Josephson junctions.

In this paper, we discuss the number of zero-energy ABS at the interface between two superconductors belonging to different topological class by solving the Bogoliubov–de Gennes equation analytically. We first study the interfacial states between two superconductors belonging to different Sato numbers. Since definition of the Sato number requires the presence of the time-reversal symmetry (TRS) of the junction, the zero-energy ABS appears only when the phase difference across the junction (φ) is 0 or π . At $\varphi = 0$ or π , we confirm that the number of the zero-energy ABSs is equal to the difference of Sato numbers in the two superconductors consistently with the bulk-boundary correspondence. We also show that the Josephson current at the zero temperature has large values near $\varphi = 0$ or π because of the resonant tunneling through ABS at the zero-energy. Next we confirmed that the number of zero-energy ABSs appearing at the interface between two different chiral superconductors is equal to

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the difference in the TKNN numbers in the two superconductors. In contrast to d_{xy} and p_x cases, the ABSs at the zero-energy do not directly affect the Josephson current between two chiral superconductors at low temperature. We also discuss the stability of π state at the Josephson junctions just below superconducting transition temperature T_c .

This paper is organized as follows. In Section 2, we discuss a theoretical model of Josephson junction consisting two topological superconductors. In Section 3, we study the interfacial ABS between two superconductors characterized by different Sato numbers. The number of the zero-energy ABS and the Josephson effect are studied for two chiral superconductors in Section 4. We summarize this paper in Section 5.

2. Model

Let us consider a Josephson junction consisting of two superconductors as shown in Fig. 1, where the electric current flows in the x direction and the junction width in the y direction is L_j . We apply the periodic boundary condition in the y direction and consider the limit of $L_j \rightarrow \infty$.

The Bogoliubov–de Gennes (BdG) Hamiltonian in momentum space reads

$$H_{\text{BdG}}(\mathbf{k}) = \begin{bmatrix} \hat{h}(\mathbf{k}) & \hat{\Delta}(\mathbf{k}) \\ -\hat{\Delta}^*(-\mathbf{k}) & -\hat{h}^*(-\mathbf{k}) \end{bmatrix}, \quad (1)$$

$$\hat{h}(\mathbf{k}) = \xi_{\mathbf{k}} \hat{\sigma}_0, \quad \xi_{\mathbf{k}} = \frac{\hbar^2 \mathbf{k}^2}{2m} - \mu, \quad (2)$$

where $\hat{\sigma}_j$ for $j=1-3$ are the Pauli matrices, $\hat{\sigma}_0$ is the unit matrix in spin space, and μ is the chemical potential. In this paper, we consider the following pair potentials $\hat{\Delta}(\gamma)$

$$\begin{array}{ll} \Delta i \hat{\sigma}_2 & \text{singlet } s, \\ \Delta 2 \cos(\gamma) \sin(\gamma) i \hat{\sigma}_2 & \text{singlet } d_{xy}, \\ \Delta \cos(\gamma) \hat{\sigma}_1 & \text{triplet } p_x, \\ \Delta e^{i n \gamma} i \hat{\sigma}_2 & \text{singlet chiral}, \\ \Delta e^{i n \gamma} \hat{\sigma}_1 & \text{triplet chiral}, \end{array} \quad (3)$$

where Δ is the amplitude of the pair potential, $-\pi/2 \leq \gamma \leq \pi/2$ is the angle between the direction of the quasiparticle's motion and the x -axis as shown in Fig. 1, $k_x = k_F \cos \gamma$ ($k_y = k_F \sin \gamma$) is the wave-number on the Fermi surface in the x (y) direction, and k_F is the Fermi wave number. The Sato number is defined for each angle γ and each spin sector in the presence of TRS. For spin-singlet superconductors, the BdG Hamiltonian in Eq. (1) is block diagonal in two Nambu space: $\mathcal{N}1$ and $\mathcal{N}2$. In $\mathcal{N}1$, spin of electron-like (hole-like) quasiparticle is \uparrow (\downarrow). On the other hand in $\mathcal{N}2$, spin of electron-like (hole-like) quasiparticle is \downarrow (\uparrow). In this paper, we assume that \mathbf{d} vector in the spin-triplet symmetry aligns along the third axis in spin space. Under this choice, the BdG Hamiltonian in Eq. (1) in the spin-triplet cases is also decoupled into $\mathcal{N}1$ and $\mathcal{N}2$. In Table 1, we summarized the Sato number $W(\gamma)$ for d_{xy} and p_x superconductors.

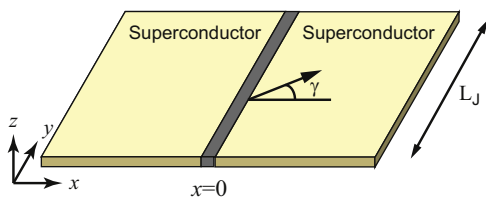


Fig. 1. A schematic picture of the Josephson junction.

Table 1

The correspondence between pairing symmetry and the Sato number W . The summary of the Sato number can be defined only in the presence of the time-reversal symmetry for each direction of wave vector on the Fermi surface γ and for each Nambu space. The Sato number of s wave case is always zero (i.e., $W_s=0$) because s -wave superconductor is topologically trivial. For spin-singlet d_{xy} symmetry, the Sato number $W_{d_{xy}}$ depends also on the Nambu space indicated by $\mathcal{N}1$ and $\mathcal{N}2$. The Sato number for spin-triplet p_x wave case W_{p_x} is always unity for all γ and the two Nambu space. Here the superconducting phase is taken to be zero.

	Angle	W_s	$W_{d_{xy}}$	W_{p_x}
$\mathcal{N}1$	$0 < \gamma < \pi/2$	0	1	1
	$-\pi/2 < \gamma < 0$	0	-1	1
$\mathcal{N}2$	$0 < \gamma < \pi/2$	0	-1	1
	$-\pi/2 < \gamma < 0$	0	1	1

In the chiral states, n in Eq. (3) must be an even integer number for spin-singlet symmetry, whereas it should be an odd integer for spin-triplet symmetry. The chiral- p , $-d$ and $-f$ wave symmetries are characterized by the TKNN number $n = \pm 1$, ± 2 and ± 3 , respectively. The TKNN number is defined in the absence of TRS. We note that the s wave superconductor is topologically trivial. Thus both the Sato number and the TKNN one are always zero in the s wave superconductor.

The energy eigen values of Eq. (1) are $E = \pm E_{\mathbf{k}, \pm}$ with $E_{\mathbf{k}, \pm} = \sqrt{\xi_{\mathbf{k}}^2 + |\Delta_{\pm}|^2}$, $\Delta_+ = \Delta(\gamma)$, and $\Delta_- = \Delta(\pi-\gamma)$. All the pair potentials in Eq. (3) satisfy $|\Delta_+| = |\Delta_-|$. In such case, the wave functions in the left and the right superconductors in $\mathcal{N}1$ are obtained as [50]

$$\Psi_L(x, y) = \hat{\Phi}_L \left[\begin{array}{l} u_L \\ v_L S_{L+}^* \end{array} \right] a e^{i k_L^x x} + \left[\begin{array}{l} v_L S_{L-} \\ u_L \end{array} \right] b e^{-i k_L^x x} + \left[\begin{array}{l} u_L \\ v_L S_{L-}^* \end{array} \right] A e^{-i k_L^x x} + \left[\begin{array}{l} v_L S_{L+} \\ u_L \end{array} \right] B e^{i k_L^x x} e^{i k_y y}, \quad (4)$$

$$\Psi_R(x, y) = \hat{\Phi}_R \left[\begin{array}{l} u_R \\ v_R S_{R+}^* \end{array} \right] C e^{i k_R^x x} + \left[\begin{array}{l} v_R S_{R-} \\ u_R \end{array} \right] D e^{-i k_R^x x} e^{i k_y y}, \quad (5)$$

$$u_j = \sqrt{\frac{1}{2} \left(1 + \frac{\Omega_j}{E} \right)}, \quad v_j = \sqrt{\frac{1}{2} \left(1 - \frac{\Omega_j}{E} \right)}, \quad \Omega_j = \sqrt{E^2 - |\Delta_j|^2}, \quad (6)$$

$$S_{j \pm} = \frac{A_{j \pm}}{|\Delta_{j \pm}|}, \quad \hat{\Phi}_j = \text{diag}\{e^{i \varphi_j / 2}, e^{-i \varphi_j / 2}\}, \quad (7)$$

$$k_j^e = \left[k_x^2 + \frac{2m}{\hbar^2} \sqrt{E^2 - |\Delta_j|^2} \right]^{1/2}, \quad k_j^h = \left[k_x^2 - \frac{2m}{\hbar^2} \sqrt{E^2 - |\Delta_j|^2} \right]^{1/2}, \quad (8)$$

where $j=L$ (R) indicates the left (right) superconductor and φ_j is the macroscopic phase of the superconductor. The coefficients A , B , C , and D are the amplitudes of outgoing waves from the interface and a and b are those of incoming waves. At the junction interface, we introduce the potential barrier described by $V_0 \delta(x)$. The boundary conditions for wave function become

$$\Psi_L(0, y) = \Psi_R(0, y), \quad (9)$$

$$-\frac{\hbar^2}{2m} \left[\frac{d}{dx} \Psi_R(x, y)_{x \rightarrow 0^+} - \frac{d}{dx} \Psi_L(x, y)_{x \rightarrow 0^-} \right] + V_0 \Psi_R(0, y) = 0. \quad (10)$$

When we calculate the energy of the interfacial ABS, we put $a = b = 0$. Since we seek the ABSs for $|E| < |\Delta_j|$, $\Psi_L(x, y)$ ($\Psi_R(x, y)$) decays at $x \rightarrow -\infty$ (∞). The decay length is approximately given by the coherence length $\xi_0 = \hbar v_F / (\pi \Delta)$ with v_F being the Fermi velocity. By using the boundary conditions in Eqs. (9) and (10), we obtain the relation among A , B , C , and D as ${}_{\mathcal{N}} \mathcal{Y} [A, B, C, D]^t = 0$, where $[\dots]^t$ is the transpose of $[\dots]$ and ${}_{\mathcal{N}} \mathcal{Y}$ is a 4×4 matrix

calculated from the boundary conditions. We define that $\Xi(E)$ is the determinant of \mathcal{Y} . It is expressed by

$$\begin{aligned} \Xi(E) &\equiv (u_L^2 - v_L^2 s_{L+} s_{L-}^*)(u_R^2 - v_R^2 s_{R+} s_{R-}^*) \\ &+ |t_n|^2 \{u_L^2 v_R^2 s_{R+}^* s_{R-} + v_L^2 s_{L+} s_{L-}^* u_R^2 \\ &- \nu e^{i\varphi} u_L v_L s_{L+} u_R v_R s_{R+}^* - \nu e^{-i\varphi} u_L v_L s_{L-}^* u_R v_R s_{R-}\}, \end{aligned} \quad (11)$$

where $\varphi = \varphi_L - \varphi_R$ ($-\pi < \varphi \leq \pi$) is the phase difference across the junction and $t_n = \cos \gamma / (\cos \gamma + iz_0)$ is the normal transmission coefficient of junction with $z_0 = V_0 / \hbar v_F$. The energy of the ABS is calculated from the condition of $\Xi(E) = 0$.

When we discuss the Josephson current, we first calculate the reflection coefficients of the junction. By eliminating C and D using the boundary conditions in Eqs. (9) and (10), we obtain a relation between (a, b) and (A, B) as

$$\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} r_{ee} & r_{eh} \\ r_{he} & r_{hh} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}, \quad (12)$$

where r_{he} and r_{eh} are the Andreev reflection coefficients and r_{ee} and r_{hh} are the normal reflection coefficients. The Josephson current is calculated based on a formula [50]

$$J = \frac{e}{2\hbar} T \sum_{\omega_n} \sum_{\gamma} \sum_{\nu} \frac{\nu}{\Omega_L} [\Delta_{L+} r_{he} - \Delta_{L-}^* r_{eh}]_{E \rightarrow i\omega_n}, \quad (13)$$

$$\begin{aligned} r_{he} &= \frac{1}{\Xi} [|t_n|^2 \{ e^{i\varphi} u_L^2 u_R v_R s_{R+}^* + e^{-i\varphi} v_L^2 s_{L+}^* s_{L-}^* u_R v_R s_{R-} \\ &- \nu u_L v_L s_{L+} v_R^2 s_{R+}^* s_{R-} - \nu u_L v_L s_{L-}^* u_R^2 \\ &+ \nu u_L v_L \{ s_{L-}^* - s_{L+}^* \} \{ u_R^2 - v_R^2 s_{R+}^* s_{R-} \}], \end{aligned} \quad (14)$$

$$\begin{aligned} r_{eh} &= \frac{1}{\Xi} [|t_n|^2 \{ e^{i\varphi} u_L^2 u_R v_R s_{R-} + e^{-i\varphi} v_L^2 s_{L-} s_{L+} u_R v_R s_{R+}^* \\ &- \nu u_L v_L s_{L-} v_R^2 s_{R+}^* s_{R-} - \nu u_L v_L s_{L+} u_R^2 \\ &+ \nu u_L v_L \{ s_{L+} - s_{L-} \} \{ u_R^2 - v_R^2 s_{R+}^* s_{R-} \}], \end{aligned} \quad (15)$$

where $\omega_n = (2n+1)\pi T$ is the Matsubara frequency. Here we explain the definition of ν appearing in Eqs. (11)–(15). When the two superconductors are in the spin-singlet symmetry, ν become 1 for the two Nambu space $\mathcal{N}1$ and $\mathcal{N}2$. This is also true when the two superconductors are in the spin-triplet symmetry. In these cases, the energies of ABS obtained from Eq. (11) are degenerate in the two Nambu space. As a result, the \sum_{ν} gives rise a factor 2 in the Josephson current in Eq. (13). When one superconductor is in the spin-singlet symmetry and the other is in the spin-triplet one, we take $\nu = 1$ in $\mathcal{N}1$ and $\nu = -1$ in $\mathcal{N}2$.

The Josephson current can be decomposed into a series of

$$J = \sum_{n=1}^{\infty} J_n \sin(n\varphi) + I_n \cos(n\varphi). \quad (16)$$

When two superconductors preserve the TRS, the coefficients I_n are usually zero. The normal transmission probability is defined by

$$T_N = \frac{1}{2} \int_{-\pi/2}^{\pi/2} d\gamma \cos \gamma |t_n|^2. \quad (17)$$

The coefficients J_n are, roughly speaking, proportional to $(T_N)^n$ for s wave Josephson junction. The lowest coupling J_1 is sensitive to the pairing symmetries of two superconductors. For instance, J_1 vanishes when one superconductor is spin-singlet and the other is spin-triplet.

3. Sato number

When the two superconductors are in the s wave symmetry, we obtain the well known results of the energy of Andreev bound

states $E^{\text{ABS}} = \pm \epsilon_{s/s}$ and the Josephson current $J_{s/s}$

$$\epsilon_{s/s} = \Delta \sqrt{1 - |t_n|^2 \sin^2(\varphi/2)}, \quad (18)$$

$$J_{s/s} = J_0 \frac{\Delta}{2\Delta_0} \frac{1}{2T_N} \int_{-\pi/2}^{\pi/2} d\gamma |t_n|^2 \cos \gamma \sin \varphi A(\epsilon_{s/s}), \quad (19)$$

$$A(\epsilon) = \frac{\Delta}{\epsilon} \tanh \left[\frac{\epsilon}{2T} \right], \quad (20)$$

$$J_0 = \frac{\pi \Delta_0}{2eR_N}, \quad \frac{1}{R_N} = \frac{2e^2}{\hbar} T_N N_c, \quad N_c = \frac{Wk_F}{\pi} \quad (21)$$

where Δ_0 is the amplitude of pair potential at $T=0$ and N_c is the number of propagating channels at the fermi level. The amplitude of the critical current at $T=0$ is J_0 in the s wave junctions [53]. There is no zero-energy ABS in the s/s junctions for $|t_n| < 1$ because the s wave superconductor is always topologically trivial.

When s wave superconductor is on the left and d_{xy} wave one is on the right, the equation for the energy of the Andreev bound states is obtained as

$$2E \sqrt{\Delta^2 - E^2} - \Delta^2 |t_n|^2 |\theta_d| \sin \varphi = 0. \quad (22)$$

Here we approximately changes $\sqrt{\Delta^2 \theta_d^2 - E^2}$ to $\sqrt{\Delta^2 - E^2}$ with $\theta_d = \sin(2\gamma)$. This approximation is possible because the presence or absence of the zero-energy ABS is sensitive only to the sign changing of pair potential on the fermi surface [28]. The energy of the ABS in $\mathcal{N}1$ and that in $\mathcal{N}2$ are identical to each other. We find that $E^{\text{ABS}} = \epsilon_{s/d_{xy}}^{\pm}$ and $\epsilon_{s/d_{xy}}^{\pm}$ with

$$\epsilon_{s/d_{xy}}^{\pm} = \Delta \sin \left(\frac{\alpha_{s/d_{xy}}}{2} \right), \quad (23)$$

$$\epsilon_{s/d_{xy}}^{\pm} = \Delta \cos \left(\frac{\alpha_{s/d_{xy}}}{2} \right) \text{sgn} \left[\sin \left(\frac{\alpha_{s/d_{xy}}}{2} \right) \right], \quad (24)$$

$$\sin(\alpha_{s/d_{xy}}) = |t_n|^2 \theta_d \sin \varphi. \quad (25)$$

Since $-\pi/2 \leq \alpha_{s/d_{xy}} \leq \pi/2$, $\epsilon_{s/d_{xy}}^{\pm}$ does not become zero. At $|t_n| \rightarrow 0$, the two superconductors are separated from each other and $\epsilon_{s/d_{xy}}^{\pm} = 0$ for all γ . Such zero-energy ABSs correspond to the surface bound states of d_{xy} wave superconductor. Namely there is one zero-energy surface state for each Nambu space for each γ , which is a result of the bulk-boundary correspondence of isolated superconductors. At $|t_n| \neq 0$, a zero-energy state appears for each γ and each Nambu space only when $\varphi = 0$ or π . In Fig. 2(a), we show $\epsilon_{s/d_{xy}}^{\pm}$ as a function of γ for several choices of φ at $z_0 = 3$. The number of the zero-energy ABSs at $\varphi = 0$ is equal to $|W_s(\gamma) - W_{d_{xy}}(\gamma)| = 1$ for each Nambu space. The zero-energy ABSs disappear for $\varphi \neq 0$ because the Sato number is not well defined in the absence of the TRS.

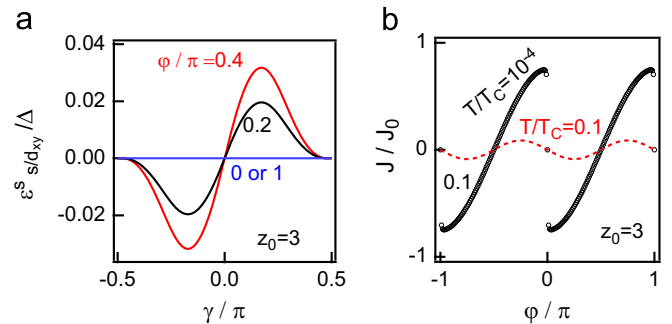


Fig. 2. The results for s/d_{xy} junctions. The energy of the Andreev bound state $\epsilon_{s/d_{xy}}^{\pm}$ is plotted as a function of γ at $z_0 = 3$ in (a). The current-phase relationship is shown in (b).

In fact, the Josephson current flows through the junction interface for $\varphi \neq 0$ as

$$J_{s/d_{xy}} = J_0 \frac{\Delta}{\Delta_0} \frac{1}{2T_N} \int_{-\pi/2}^{\pi/2} d\gamma \frac{\cos \gamma |t_n|^4 \theta_d^2 \sin(2\varphi)}{4 \cos(\alpha_{s/d_{xy}})} \times [A(e_{s/d_{xy}}^c) - A(e_{s/d_{xy}}^s)]. \quad (26)$$

The current-phase relationship (CPR) for $T \lesssim T_c$ becomes

$$J_{s/d_{xy}} = -\sin(2\varphi) \times J_0 \frac{\Delta}{\Delta_0} \left(\frac{\Delta}{T}\right)^3 \frac{1}{2T_N} \int_{-\pi/2}^{\pi/2} d\gamma \frac{\cos \gamma |t_n|^4 \theta_d^2}{96}. \quad (27)$$

The lowest order coupling is absent (i.e., $J_1 = 0$) because s and d_{xy} wave pairing function are orthogonal to each other. At $T=0$, the Josephson current becomes

$$J_{s/d_{xy}} = -\cos(\varphi) \operatorname{sgn}[\sin(\varphi)] \times J_0 \frac{1}{2T_N} \int_{-\pi/2}^{\pi/2} d\gamma \frac{\cos \gamma |t_n|^2 |\theta_d|}{\sqrt{1 + |t_n|^2 |\theta_d| \sin(\varphi)}}. \quad (28)$$

The resonant transmission through the zero-energy ABSs mainly contributes to the Josephson current at $T=0$. In fact, the Josephson current becomes large at $\varphi = 0$ and π , and jumps as shown in Fig. 2 (b), where we plot the Josephson current as a function of φ . We fix z_0 at 3, which leads to the normal transmission probability $T_N \approx 0.07$. At $T=0$, the amplitude of the Josephson current is roughly proportional to T_N as shown in Eq. (28), which is a result of the resonant transmission through the ABS at the zero-energy.

The similar conclusion is obtained when we replace the spin-singlet d_{xy} wave superconductor by the spin-triplet p_x wave one. The energy of ABS is obtained as $E^{\text{ABS}} = e_{s/p_x}^s$ and e_{s/p_x}^c . These energies are given by

$$e_{s/p_x}^s = \Delta \sin\left(\frac{\alpha_{s/p_x}}{2}\right), \quad (29)$$

$$e_{s/p_x}^c = \Delta \cos\left(\frac{\alpha_{s/p_x}}{2}\right) \operatorname{sgn}\left[\sin\left(\frac{\alpha_{s/p_x}}{2}\right)\right], \quad (30)$$

$$\sin(\alpha_{s/p_x}) = \nu |t_n|^2 \theta_p \sin \varphi, \quad (31)$$

with $\theta_p = \cos \gamma$. We note that there are four dispersion branches in the ABS because of the contributions from two Nambu space. The zero-energy ABS appears only when $\varphi = 0$ or π . The Josephson current in this junction is given by Eq. (26) with $\theta_d \rightarrow \theta_p$.

As shown in Table 1, d_{xy} wave and p_x wave symmetries are classified into the same Sato number for $0 < \gamma < \pi/2$ in $\mathcal{N}1$ and $-\pi/2 < \gamma < 0$ in $\mathcal{N}2$. In these cases, ABSs may not appear at the zero-energy. On the other hand, two zero-energy ABSs are expected for $0 < \gamma < \pi/2$ in $\mathcal{N}2$ and $-\pi/2 < \gamma < 0$ in $\mathcal{N}1$ because of $|W_{d_{xy}} - W_{p_x}| = 2$. These prediction can be confirmed by considering the equation for energy of the ABS

$$E^2 - \frac{|t_n|^2}{2} \left[E^2 + \sqrt{\Delta^2 \theta_d^2 - E^2} \sqrt{\Delta^2 \theta_p^2 - E^2} + \nu \Delta^2 \theta_p \theta_d \cos(\varphi) \right] = 0, \quad (32)$$

where $\nu = 1$ for $\mathcal{N}1$ and $\nu = -1$ for $\mathcal{N}2$. By substituting $E=0$, we find $|\theta_d| + \nu \theta_d \cos \varphi = 0$. Therefore, at $\varphi = 0$, the zero-energy ABS appear for $0 < \gamma < \pi/2$ in $\mathcal{N}2$, and for $-\pi/2 < \gamma < 0$ in $\mathcal{N}1$. To have expressions of E^{ABS} , we estimate the term $\sqrt{\Delta^2 \theta_d^2 - E^2} \sqrt{\Delta^2 \theta_p^2 - E^2} < \Delta^2 |\theta_p \theta_d| - E^2$. Mathematically this approximation is justified for $\gamma \approx \pi/6$. The sign changing of the pair potential is responsible for the formation of ABS, which has already been taken into account in Eq. (32). Although the approximation slightly modifies the γ dependence of the energy of ABS, it is not essential in the following argument. As a result, we obtain $E^{\text{ABS}} = \pm e_{d_{xy}/p_x}^c$ and

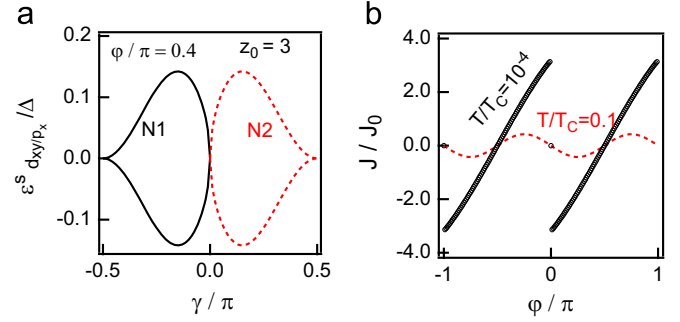


Fig. 3. The results for d_{xy}/p_x junctions at $z_0 = 3$. The energy of the Andreev bound state $\pm e_{d_{xy}/p_x}^s$ is plotted as a function of γ in (a). The current-phase relationship of the Josephson current is shown in (b), where results for $T = 0.1T_c$ are amplified by 10.

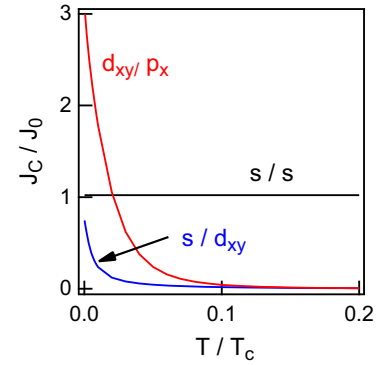


Fig. 4. The maximum value of the Josephson current is plotted as a function of temperature. Here we choose $z_0 = 3$.

$\pm e_{d_{xy}/p_x}^s$ with

$$e_{d_{xy}/p_x}^c = \Delta |t_n| \sqrt{|\theta_p \theta_d|} \cos(\varphi/2) \theta(\nu \theta_d), \quad (33)$$

$$e_{d_{xy}/p_x}^s = \Delta |t_n| \sqrt{|\theta_p \theta_d|} \sin(\varphi/2) \theta(-\nu \theta_d), \quad (34)$$

where $\theta(x)$ is the step function: $\theta = 1$ for $0 < x$ and $\theta = 0$ otherwise. In Fig. 3(a), we show $\pm e_{d_{xy}/p_x}^s$ at $z_0 = 3$ and $\varphi = 0.4\pi$. When we consider $\varphi \rightarrow 0$, there are doubly degenerate ABSs at the zero-energy for $-\pi/2 < \gamma < 0$ in $\mathcal{N}1$ and for $0 < \gamma < \pi/2$ in $\mathcal{N}2$. The doubly degenerate dispersionless ABSs at the zero-energy drastically affect the Josephson current which is calculated to be

$$J_{d_{xy}/p_x} = J_0 \frac{\Delta}{\Delta_0} \sum_{\nu} \frac{1}{2T_N} \int_{-\pi/2}^{\pi/2} d\gamma \frac{\cos \gamma |t_n|^2 |\theta_d \theta_p| \sin \varphi}{2} \times [A(e_{d_{xy}/p_x}^c) - A(e_{d_{xy}/p_x}^s)]. \quad (35)$$

The expression of the Josephson current is given by

$$J_{d_{xy}/p_x} = -\sin(2\varphi) \times J_0 \frac{\Delta}{\Delta_0} \left(\frac{\Delta}{T}\right)^3 \frac{1}{2T_N} \int_{-\pi/2}^{\pi/2} d\gamma \frac{\cos \gamma |t_n|^4 \theta_d^2 \theta_p^2}{96}, \quad (36)$$

for $T \lesssim T_c$. The lowest coupling vanishes because one superconductor is spin-singlet and the other is spin-triplet (i.e., $J_1 = 0$ in Eq. (16)). At $T=0$, we find

$$J_{d_{xy}/p_x} = [\sin(\varphi/2) - \cos(\varphi/2) \operatorname{sgn}(\varphi)] \times J_0 \frac{1}{2T_N} \int_{-\pi/2}^{\pi/2} d\gamma \cos \gamma |t_n| \sqrt{|\theta_d \theta_p|}. \quad (37)$$

In Fig. 3(b), we plot the Josephson current as a function of φ at $z_0 = 3$. At very low temperature, the Josephson current shows unusual current-phase relationship and has large amplitude proportional to $\sqrt{T_N}$ around $\varphi = 0$. At $\varphi = \pi$, $\pm e_{d_{xy}/p_x}^c$ describes the

doubly degenerate dispersionless ABSs at the zero-energy which also contribute to the large Josephson current as shown in Fig. 3 (b). Tuning of φ at π is equivalent to the changing the sign of $W_{d_{xy}}$ in Table 1. In Fig. 4, we show the critical Josephson current as a function of temperature for $0 \leq T \leq 0.2T_c$ with $z_0 = 3$. In this temperature range, the maximum value of $J_{S/S}$ is saturates at J_0 . The results for $J_{S/d_{xy}}$ and J_{d_{xy}/P_x} are much smaller than J_0 for $T > 0.1T_c$. They increase rapidly with decreasing temperature for $T < 0.1T_c$. Such effect is known as the low-temperature anomaly of the Josephson current [29,33].

4. TKNN number

In this section, we consider a junction which consists of two chiral superconductors, where the TKNN number in the left superconductor is n and that in the right one is m . We can calculate the energy of ABS and the Josephson current without any further approximations because the amplitude of the pair potential $|\Delta(\gamma)|$ is independent of γ and TKNN numbers. We first show the results of the energy of ABS and the Josephson current. Secondly we count the number of zero-energy ABSs of the junction. Then we discuss properties of the Josephson current.

We show the results for a junction which consists of two spin-singlet chiral superconductors. In this case, the two TKNN numbers (n in the left superconductors and m in the right one) are even integer numbers. The energy of ABSs is obtained as $E^{\text{ABS}} = \epsilon_{\pm}^{ee}$ with

$$\epsilon_{\pm}^{ee} = \pm \Delta \text{sgn}[\sin(X_{\pm}^{ee})] \cos(X_{\pm}^{ee}), \quad (38)$$

$$X_{\pm}^{ee} = \frac{\alpha^{ee} \pm (n-m)\gamma}{2}, \quad (39)$$

$$\cos \alpha^{ee} = (1-|t_n|^2) \cos\{(n+m)\gamma\} + |t_n|^2 \cos \varphi, \quad (40)$$

where $0 \leq \alpha^{ee} \leq \pi$. The Josephson current J_{ee} is also expressed in terms of the energy of ABS

$$J_{ee} = J_0 \frac{\Delta}{\Delta_0} \frac{1}{2T_N} \int_{-\pi/2}^{\pi/2} d\gamma \frac{\cos(\gamma) |t_n|^2 \sin \varphi}{2 \sin \alpha^{ee}} \times [\sin(2X_+^{ee})A(\epsilon_+^{ee}) + \sin(2X_-^{ee})A(\epsilon_-^{ee})]. \quad (41)$$

Next we show the results for a junction where the two superconductors belong to the spin-triplet chiral states. The two TKNN numbers (n in the left superconductors and m in the right one) are odd integer numbers. We find the energy of ABS as $E^{\text{ABS}} = \epsilon_{\pm}^{oo}$ with

$$\epsilon_{\pm}^{oo} = \mp \Delta \text{sgn}[\cos(X_{\pm}^{oo})] \sin(X_{\pm}^{oo}), \quad (42)$$

$$X_{\pm}^{oo} = \frac{\alpha^{oo} \pm (n-m)\gamma}{2}, \quad (43)$$

$$\cos \alpha^{oo} = (1-|t_n|^2) \cos\{(n+m)\gamma\} - |t_n|^2 \cos \varphi. \quad (44)$$

The Josephson current is calculated as

$$J_{oo} = J_0 \frac{\Delta}{\Delta_0} \frac{1}{2T_N} \int_{-\pi/2}^{\pi/2} d\gamma \frac{\cos(\gamma) |t_n|^2 \sin \varphi}{2 \sin \alpha^{oo}} \times [\sin(2X_+^{oo})A(\epsilon_+^{oo}) + \sin(2X_-^{oo})A(\epsilon_-^{oo})]. \quad (45)$$

Finally we show the results for a junction where the spin-singlet chiral superconductor occupies the left hand side of the junction and the spin-triplet chiral superconductor occupies the right hand side. The TKNN numbers in the left superconductor n is an even integer and that in the right one m is an odd integer number. We find the energy of ABS is $E^{\text{ABS}} = \epsilon_{\pm}^{eo}$ with

$$\epsilon_{\pm}^{eo} = \mp \Delta \text{sgn}[\cos(X_{\pm}^{eo})] \sin(X_{\pm}^{eo}), \quad (46)$$

$$X_{\pm}^{eo} = \frac{\alpha_v^{eo} \pm \{(n-m)\gamma - \pi/2\}}{2}, \quad (47)$$

$$\cos \alpha_v^{eo} = (1-|t_n|^2) \sin\{(n+m)\gamma\} + |t_n|^2 \nu \sin \varphi. \quad (48)$$

The Josephson current becomes

$$J_{eo} = J_0 \frac{\Delta}{\Delta_0} \sum_{\nu = \pm 1} \frac{1}{2T_N} \int_{-\pi/2}^{\pi/2} d\gamma \frac{\cos(\gamma) |t_n|^2 \nu \cos \varphi}{4 \sin \alpha_v^{eo}} \times [\sin(2X_+^{eo})A(\epsilon_+^{eo}) + \sin(2X_-^{eo})A(\epsilon_-^{eo})], \quad (49)$$

where $\nu = \pm 1$ indicates two Nambu space.

From the expression of the bound state energy, it is easy to count the number of ABSs at the zero-energy. For example, we consider the junction with both n and m being odd integers here. At $|t_n| = 0$, the energies of the bound state become

$$\epsilon_+^{eo} = -\Delta \text{sgn}(\cos n\gamma) \sin n\gamma, \quad (50)$$

$$\epsilon_-^{eo} = \Delta \text{sgn}(\cos m\gamma) \sin m\gamma, \quad (51)$$

because of $\alpha^{oo} = (n+m)\gamma$. They represent the dispersion of the surface bound states when the two superconductors are separated from each other. It is easy to confirm that the number of the zero-energy surface states of ϵ_+^{eo} is $|n|$ and that of ϵ_-^{eo} is $|m|$, which is a result of the bulk-boundary correspondence for an isolated superconductor. At finite $|t_n|$, the solutions of $\epsilon_{\pm}^{eo} = 0$ in Eq. (42) require the relation

$$\tan\left\{\frac{(n-m)\gamma}{2}\right\} = \pm \tan\left(\frac{\alpha_{oo}}{2}\right). \quad (52)$$

The left hand side of Eq. (52) goes positive infinity $|n-m|/2$ times and goes negative infinity $|n-m|/2$ times within the interval of $-\pi/2 \leq \gamma \leq \pi/2$. Since $0 \leq \alpha_{oo} \leq \pi$, the right hand side of Eq. (52) $\tan(\alpha_{oo}/2)$ remains positive value for all γ . Thus the number of the solutions of Eq. (52) is $|n-m|$ which corresponds to the difference in the TKNN numbers between the two superconductors. In Fig. 5, we plot the left hand side of Eq. (52) with a solid line as a function of γ and the right hand side with two broken lines. In Fig. 5(a), we choose $n=1$, $m=-3$, $z_0=0.5$, and $\varphi=0.2\pi$. The solid lines and two broken lines cross four times, which means four the zero-energy

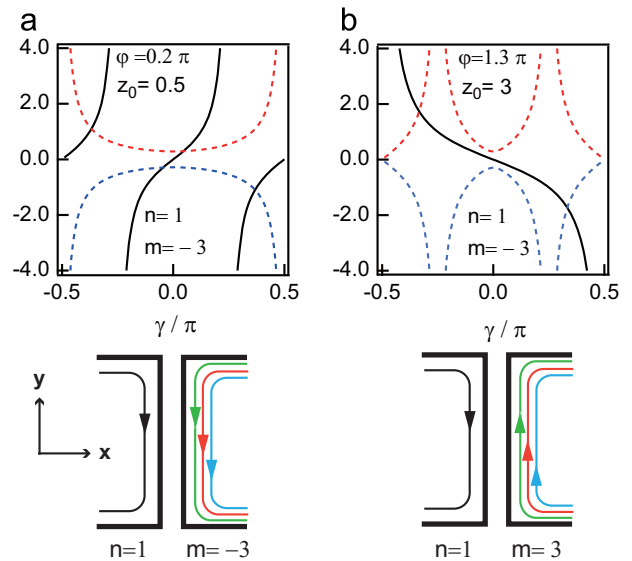


Fig. 5. The left- and right-hand side of Eq. (52) are plotted with a solid line and two broken lines, respectively. We choose $n=1$, $m=-3$, $z_0=0.5$, and $\varphi=0.2\pi$ in (a). The parameters in (b) are $n=1$, $m=3$, $z_0=3$, and $\varphi=1.3\pi$.

ABSs (i.e., $4 = |1 - (-3)|$). The number of the solutions independent of junction parameters such as z_0 and φ . In the lower panel in Fig. 5 (a), we illustrate the schematic picture of the chiral edge states at $|t_n| = 0$. The arrows indicate the direction of the chiral current. In Fig. 5(b), we choose $n=1$, $m=3$, $z_0=3$, and $\varphi=1.3\pi$. The results show that there are only two solutions (i.e., $2 = |1-3|$). In the lower panel, there are four chiral edges at $|t_n| = 0$. The chiral current in the left superconductor flows the opposite direction to that in the right superconductor in this case. For finite $|t_n|$, two chiral edge currents opposite directions are cancel each other out and two ABS remain at the zero-energy. Therefore we confirmed that the number of the ABS at the zero-energy is $|n-m|$ for each Nambu space when the TKNN number of two superconductors are n and m . The similar argument can be applied to Eqs. (38) and (46). The presence of $|n-m|$ zeros has been suggested by the index theorem [51,52]. Here we count the number of ZES by solving Eq. (11) explicitly.

The Josephson current in Eqs. (41) and (45) has a common expression near T_c

$$J = J_0 \frac{\Delta}{\Delta_0} \frac{\Delta}{T} \sin \varphi \frac{1}{4T_N} I_{(n-m)}(z_0), \quad (53)$$

$$I_{n-m}(z_0) = \int_{-\pi/2}^{\pi/2} d\gamma \cos(\gamma) |t_n|^2 \cos\{(n-m)\gamma\} \quad (54)$$

where n and m are both even integer numbers or both odd ones. The junction is stable at π phase difference in the case of $I_{n-m} < 0$. At $z_0 = 0$, the integral for highly transparent junctions becomes

$$I_{n-m}(z_0 = 0) = \frac{2}{1-(n-m)^2} (-1)^{(n-m)/2}. \quad (55)$$

On the other hand in the tunneling limit $z_0 \gg 1$, we find

$$I_{n-m}(z_0 \gg 1) = \frac{4}{z_0^2} \frac{(-1)^{(n-m)/2}}{\{(n-m)^2 - 1\} \{(n-m)^2 - 9\}}. \quad (56)$$

In Table 2, we indicate the stable state of the Josephson junctions near T_c . At $n-m=4$, π -state is stable in the highly transparent limit and 0-state is stable in the tunneling limit. Therefore the junction undergoes the transition from π -state to 0-state when we decrease the transparency of the junction. Such junction can be realized with chiral- d wave superconductor.

In contrast to Section 3, the ABSs at the zero-energy do not directly affect the Josephson current at low temperature. At $T=0$, Eq. (41) becomes

$$J_{ee} = J_0 \frac{\sin \varphi}{2T_N} \int_{-\pi/2}^{\pi/2} d\gamma \frac{\cos(\gamma) |t_n|^2}{2 \sin \alpha^{ee}} \times \left[\frac{\sin(2X_+^{ee})}{|\cos(X_+^{ee})|} + \frac{\sin(2X_-^{ee})}{|\cos(X_-^{ee})|} \right]. \quad (57)$$

Mathematically speaking, the zeros in the $|\cos(X_{\pm}^{ee})|$ are cancelled by the numerator. Instead of the zeros of E^{ABS} , minima of $\sin \alpha^{ee}$ in the denominator determine the amplitude of Josephson current at low temperature. This property may come from the fact that the ABSs have the dispersion as a function of γ . The Josephson current shows the logarithmic dependence of temperature at intermediate

Table 2

The stable states of the Josephson junction near T_c are indicated by '0' or ' π '. Here n and m are both even integer numbers or both odd integer numbers. As a consequence, $n-m$ becomes an even integer.

$n-m$	0	2	4	6	8	10
$z_0 = 0$	0	0	π	0	π	0
$z_0 \gg 1$	0	0	0	π	0	π

temperature region between $T=0$ and $T=T_c$ [36,38]. This argument can be applied also to the Josephson current in Eq. (45).

When n is an even integer and m is an odd integer, the Josephson current in Eq. (49) near T_c becomes

$$J_{eo} = -\sin(2\varphi) J_0 \frac{\Delta}{\Delta_0} \left(\frac{\Delta}{T}\right)^3 \times \frac{1}{2T_N} \int_{-\pi/2}^{\pi/2} d\gamma \frac{\cos(\gamma) |t_n|^4 \cos\{2(n-m)\gamma\}}{96}. \quad (58)$$

The coefficient proportional to $-\sin(2\varphi)$ is positive in the limit of both $z_0 = 0$ and $z_0 \gg 1$.

In Fig. 6, we show the maximum value of the Josephson current J_c for $n=2$ and $z_0=5$, where m is the TKNN number on the right superconductor. The two superconductors belong to spin-singlet symmetry. The results for $m=-4, -2, 0$, and 2 in Fig. 6 (a) monotonically increase with decreasing temperature. On the other hand, the results for $m=4, 6$, and 8 in Fig. 6(b) show the nonmonotonic temperature dependence. In particular, the results for $m=6$ indicate the transition from 0 state to π state with the decrease of temperature around $T=0.25T_c$. We also note that the π state is stable for $m=-4$ and 8 all temperature range. The nonmonotonic behavior of J_c indicates the instability of the junction between 0 and π states. For $m=2$, the Josephson current takes its maximum at $\varphi=0.55\pi$ at $T=0$. In another cases, CPR is almost sinusoidal. The amplitudes of J_c become smaller for larger $|n-m|$ because the integrand of Eq. (57) oscillates more frequently as a function of γ for larger $|n-m|$.

In Fig. 7, we show the maximum value of the Josephson current J_c for $n=1$ and $z_0=5$, where m is the TKNN number on the right superconductor. The results for $m=-3, -1, 1$, and 3 in (a) monotonically increase with decreasing temperature. However,

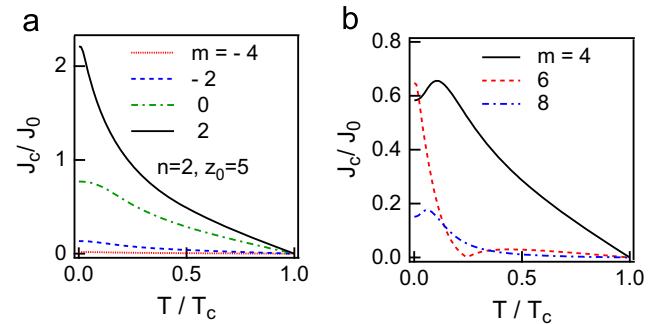


Fig. 6. The Josephson critical current is plotted as a function of temperature for $z_0=5$. We fix TKNN number at $n=2$ in the left superconductor. The TKNN number in the right superconductor is m .

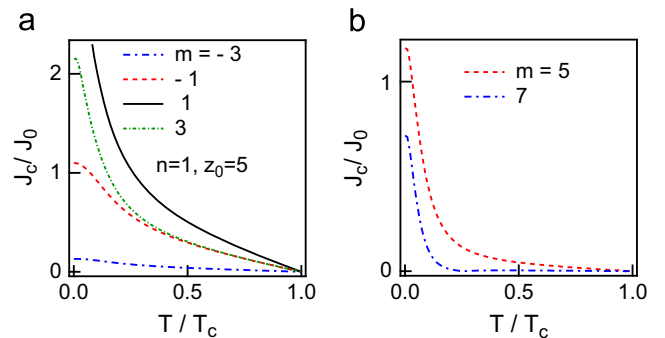


Fig. 7. The Josephson critical current is plotted as a function of temperature for $z_0=5$. We fix TKNN number at $n=1$ in the left superconductor. The TKNN number in the right superconductor is m .

the results for $m=7$ show the nonmonotonic dependence on temperature because of the $0-\pi$ transition around $T=0.2T_c$. Consistently with Table 2, the 0 state is stable at $m \neq 7$ and $z_0 \gg 1$ all temperature range.

5. Conclusion

We have studied the properties of the Andreev bound states (ABSs) appearing at the junction interface between two unconventional superconductors. We consider superconductors characterized by two types of topological numbers: the Sato number in Section 3 and the TKNN number in Section 4. We confirmed that the number of the ABSs at the zero-energy is identical to the difference of topological numbers in the two superconductors. We also discuss the effects of the ABSs at the zero-energy on the Josephson current. The zero-energy ABSs directly contribute to the low-temperature anomaly of the Josephson current when the two superconductors characterized by the Sato number. On the other hand, ABSs at the zero-energy do not directly affect the Josephson current when the two superconductors characterized by the TKNN number. In the latter case, we also found the $0-\pi$ transition as a function of the transparency of the junction and temperature.

Acknowledgments

This work was supported by KAKENHI (No. 22540355) and the “Topological Quantum Phenomena” (No. 22103002) Grant-in Aid for Scientific Research on Innovative Areas from the Ministry of Education, Culture, Sports, Science and Technology (MEXT) of Japan. Y.A. is grateful to Y. Tanaka and M. Sato for useful discussion.

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