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Nonmonotonic temperature dependence of critical current in diffusive *d*-wave junctions

T. Yokoyama,[1] Y. Tanaka,[1] A. A. Golubov,[2] and Y. Asano[3]
[1]*Department of Applied Physics, Nagoya University, Nagoya 464-8603, Japan and CREST, Japan Science and Technology Corporation (JST), Nagoya 464-8603, Japan*
[2]*Faculty of Science and Technology, University of Twente, 7500 AE, Enschede, The Netherlands*
[3]*Department of Applied Physics, Hokkaido University, Sapporo 060-8628, Japan*

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We study the Josephson effect in D/I/DN/I/D junctions, where I, DN, and D denote an insulator, a diffusive normal metal, and a *d*-wave superconductor, respectively. The Josephson current is calculated based on the quasiclassical Green's function theory with a general boundary condition for unconventional superconducting junctions. In contrast to *s*-wave junctions, the product of the Josephson current and the normal-state resistance is enhanced by making the interface barriers stronger. The Josephson current has a nonmonotonic temperature dependence due to the competition between the proximity effect and the midgap Andreev resonant states.

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Since the discovery of the Josephson effect[1] in superconductor/insulator/superconductor (SIS) junctions, it has been studied in various types of structures.[2,3] The Josephson current flows not only through thin insulators but also through normal metals or ferromagnets.[1–4] In superconductor/diffusive normal metal/superconductor (S/DN/S) junctions, Josephson current is carried by Cooper pairs penetrating into the DN as a result of the proximity effect. In *s*-wave superconductor junctions, the maximum amplitude of Josephson current (I_C) monotonically increases with the decrease of temperature.[5–7] This is a natural consequence of the fact that interference effects are stronger at lower temperatures. The Josephson effect, however, depends strongly on pairing symmetries of superconductors because it is an essentially phase sensitive phenomenon. In *d*-wave superconductor/insulator/*d*-wave superconductor (DID) junctions, for instance, I_C first increases with the decrease of temperature, then decreases, and can even change its sign at low temperatures for certain orientations.[8–11] This nonmonotonic behavior of Josephson current is caused by 0-π transition due to the midgap Andreev resonant states (MARS) forming at junction interfaces.[12,13] A similar effect is also observed in Josephson current through ferromagnets.[4] Thus the nonmonotonic temperature dependence of I_C is a sign of unusual interference effect.

The quasiclassical Green's function theory is a powerful tool to study quantum transport in superconducting junctions. The circuit theory[14] provides the boundary conditions for the Usadel equations[15] widely used in diffusive superconducting junctions. These boundary conditions generalize the Kupriyanov-Lukichev conditions[6] for an arbitrary type of connector which couples the diffusive nodes. Recently Tanaka *et al.*[16–18] have extended the circuit theory[14] to systems with unconventional superconductors. An application of the extended circuit theory to DN/*d*-wave superconductor junctions has revealed a strong competition (destructive interference) between the MARS and the proximity effect in DN (Refs. 16 and 17). This competition, however, has not yet tested experimentally. Thus it is important to propose the way to verify this prediction.

In the present paper, we show that the Josephson effect is a suitable tool to observe the above competition. We extend the previous theory[16,17] in order to treat the external phase difference between the left and right superconductors. Applying this formalism, we calculate Josephson current in *d*-wave superconductor/insulator/diffusive normal metal/insulator/*d*-wave superconductor (D/I/DN/I/D) junctions, solving the Usadel equations with new boundary conditions. This allows us to study the influences of the proximity effect and the formation of MARS on the Josephson current simultaneously. We find that the competition between the proximity effect and the formation of MARS provides a new mechanism for a nonmonotonic temperature dependence of the maximum Josephson current.

We consider a junction consisting of *d*-wave superconducting reservoirs (D) connected by a quasi-one-dimensional diffusive conductor (DN) with a resistance R_d and a length L much larger than the mean free path. The DN/D interface located at $x=0$ has the resistance R'_b, while the DN/D interface located at $x=L$ has the resistance R_b. We model infinitely narrow insulating barriers by the delta function $U(x)=H\delta(x-L)+H'\delta(x)$. The resulting transparencies of the junctions T_m and T'_m are given by $T_m=4\cos^2\phi/(4\cos^2\phi+Z^2)$ and $T'_m=4\cos^2\phi/(4\cos^2\phi+Z'^2)$, where $Z=2H/v_F$ and $Z'=2H'/v_F$ are dimensionless constants and ϕ is the injection angle measured from the interface normal to the junction and v_F is Fermi velocity. We show the schematic illustration of the model in Fig. 1.

We parametrize the quasiclassical Green's functions G and F with a function Φ_ω (Refs. 2 and 3),

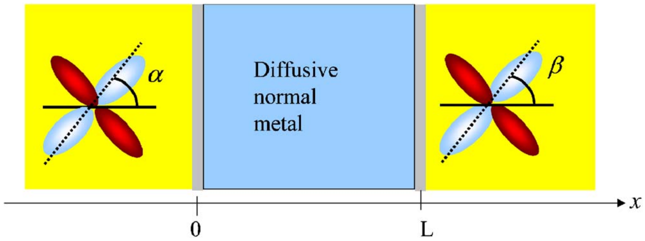

FIG. 1. (Color online) Schematic illustration of the model.

$$G_\omega = \frac{\omega}{\sqrt{\omega^2 + \Phi_\omega \Phi_{-\omega}^*}}, \quad F_\omega = \frac{\Phi_\omega}{\sqrt{\omega^2 + \Phi_\omega \Phi_{-\omega}^*}}, \tag{1}$$

where ω is the Matsubara frequency. Then the Usadel equation reads[15]

$$\xi^2 \frac{\pi T_C}{\omega G_\omega} \frac{\partial}{\partial x}\left(G_\omega^2 \frac{\partial}{\partial x} \Phi_\omega \right) - \Phi_\omega = 0 \tag{2}$$

with the coherence length $\xi = \sqrt{D/2\pi T_C}$, the diffusion constant D, and the transition temperature T_C. From the conservation law for the matrix current,[17] the boundary conditions are given by

$$\frac{G_\omega}{\omega}\frac{\partial}{\partial x}\Phi_\omega = -\frac{R_d}{R_b' L}\left(-\frac{\Phi_\omega}{\omega} I_1' + e^{-i\varphi}(I_2' - i I_3') \right),$$

$$I_1' = \left\langle \frac{2T_m' g_S'}{A'} \right\rangle', \quad I_2' = \left\langle \frac{2T_m' \bar{f}_S'}{A'} \right\rangle', \quad I_3' = \left\langle \frac{2T_m' f_S'}{A'} \right\rangle',$$

$$A' = 2 - T_m' + T_m'[g_S' G_\omega + \bar{f}_S'(B' \cos\varphi + C' \sin\varphi) + f_S'(C' \cos\varphi - B' \sin\varphi)],$$

$$B' = \frac{G_\omega}{2\omega}(\Phi_\omega + \Phi_{-\omega}^*), \quad C' = \frac{i G_\omega}{2\omega}(\Phi_\omega - \Phi_{-\omega}^*),$$

$$g_S' = \frac{g_+ + g_-}{1 + g_+ g_- + f_+ f_-},$$

$$\bar{f}_S' = \frac{f_+ + f_-}{1 + g_+ g_- + f_+ f_-}, \quad f_S' = \frac{i(f_+ g_- - f_- g_+)}{1 + g_+ g_- + f_+ f_-}, \tag{3}$$

$g_\pm = \omega / \sqrt{\omega^2 + \Delta_\pm^2}$, $f_\pm = \Delta_\pm / \sqrt{\omega^2 + \Delta_\pm^2}$, with $\Delta_\pm = \Delta(T)\cos(2\phi \mp 2\alpha)$ at $x=0$, and

$$\frac{G_\omega}{\omega}\frac{\partial}{\partial x}\Phi_\omega = \frac{R_d}{R_b L}\left(-\frac{\Phi_\omega}{\omega} I_1 + (I_2 - i I_3) \right) \tag{4}$$

at $x=L$, where I_1, I_2, and I_3 are defined similarly to I_1', I_2', and I_3' by removing the superscript "′," exchanging subscript "+" for subscript "−," putting $\varphi=0$, and substituting β for α, respectively. Here φ is the external phase difference across the junctions, and α and β denote the angles between the normal to the interface and the crystal axes of d-wave superconductors for $x \leqslant 0$ and $x \geqslant L$, respectively.

These boundary conditions derived above are quite general since with a proper choice of $\Delta_\pm$ they are applicable to any unconventional superconductor with $S_z=0$ in a time reversal symmetry conserving state. Here, S_z denotes the z component of the total spin of a Cooper pair.

The average over the various angles of injected particles at the interface is defined as

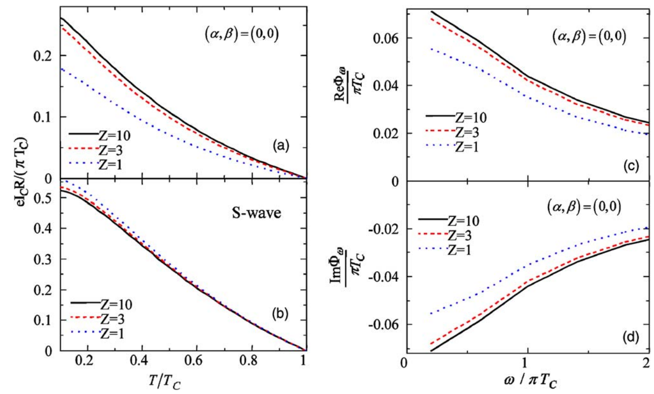

FIG. 2. (Color online) (a) Temperature dependence of $I_C R$ for $(\alpha,\beta)=(0,0)$. (b) The results for s-wave junctions for comparison. Real (c) and imaginary (d) parts of Φ_ω as a function of ω. Here we choose $R_d/R_b=1$ and $E_{Th}/\Delta(0)=1$.

$$\langle B(\phi)\rangle^{(\prime)} = \frac{\int_{-\pi/2}^{\pi/2} d\phi \cos\phi B(\phi)}{\int_{-\pi/2}^{\pi/2} d\phi T^{(\prime)}(\phi)\cos\phi} \tag{5}$$

with $B(\phi)=B$ and $T^{(\prime)}(\phi)=T_m^{(\prime)}$. It is important to note that the solution of the Usadel equation is invariant under the transformation $\alpha \to -\alpha$ or $\beta \to -\beta$. This is clear by replacing ϕ with $-\phi$ in the angular averaging. The resistance of the interface $R_b^{(\prime)}$ is given by

$$R_b^{(\prime)} = R_0^{(\prime)}\left(2 \Big/ \int_{-\pi/2}^{\pi/2} d\phi T^{(\prime)}(\phi)\cos\phi \right). \tag{6}$$

Here, for example, $R_b^{(\prime)}$ denotes R_b or R_b', and $R_0^{(\prime)}$ is the Sharvin resistance, which in the three-dimensional case is given by $R_0^{(\prime)-1} = e^2 k_F^2 S_c^{(\prime)}/(4\pi^2)$, where k_F is the Fermi wave vector and $S_c^{(\prime)}$ is the constriction area.

The Josephson current is given by

$$\frac{eIR}{\pi T_C} = i\frac{RTL}{2R_d T_C}\sum_\omega \frac{G_\omega^2}{\omega^2}\left(\Phi_\omega \frac{\partial}{\partial x}\Phi_{-\omega}^* - \Phi_{-\omega}^* \frac{\partial}{\partial x}\Phi_\omega \right) \tag{7}$$

with temperature T and $R \equiv R_d + R_b + R_b'$. In the following we focus on the $I_C R$ value where I_C denotes the magnitude of the maximum Josephson current. We consider symmetric junctions with $R_b = R_b'$ and $Z=Z'$ for simplicity.

In Fig. 2(a), $I_C R$ is plotted as a function of temperature for $R_d/R_b=1$, $E_{Th}/\Delta(0)=1$, and $(\alpha,\beta)=(0,0)$. The results show that $I_C R$ increases with the decrease of T as it does in s-wave junctions as shown in Fig. 2(b). Amplitude of $I_C R$ increases with the increase of Z in d-wave junctions, whereas the opposite tendency can be seen in s-wave junctions. The sign change of pair potential is responsible for the Z dependence of $I_C R$ in (a). In the d-wave symmetry with $\alpha=\beta=0$, injection angles of a quasiparticle can be divided into two regions: $\phi_+ = \{\phi | 0 \leqslant |\phi| < \pi/4\}$ and $\phi_- = \{\phi | \pi/4 \leqslant |\phi| \leqslant \pi/2\}$. The sign of pair potential for ϕ_+ and that for ϕ_- are opposite to each other. In general, such a sign change of pair

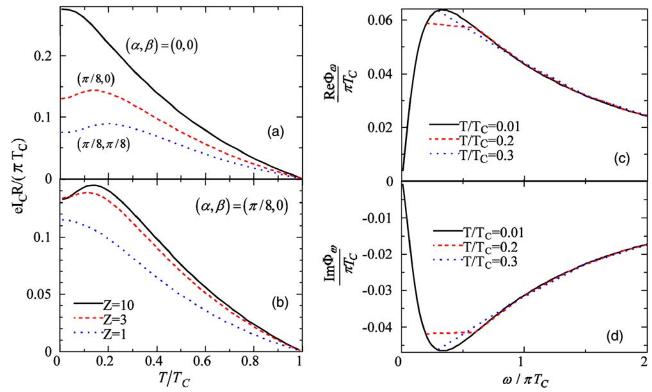

FIG. 3. (Color online) Temperature dependence of (a) I_CR at $Z=10$ for several orientation angles. (b) I_CR at $(\alpha,\beta)=(\pi/8,0)$ for several Z values. Real and imaginary parts of Φ_ω as a function of ω for $(\alpha,\beta)=(\pi/8,0)$ and $\varphi=\pi/2$ at $x=L/2$ are shown in (c) and (d), respectively. Here we choose $R_d/R_b=1$ and $E_{Th}/\Delta(0)=1$.

potentials leads to the suppression of the proximity effect in DN and hence Josephson currents. This is because Cooper pairs originated from the positive part of pair potentials cancel those originated from the negative part of pair potentials. For small Z, a quasiparticle with ϕ_+ and that with ϕ_- give opposite contribution to the proximity effect. In the actual calculation, the cancellation due to the sign change of the pair potential reduces the magnitudes of I_2, I_3, I_2', and I_3'. On the other hand, for large Z, integration over ϕ_+ has the dominant contribution to I_2, I_3, I_2', and I_3'. As a result, the cancellation of Cooper pairs becomes negligible in low transparent junctions. Thus I_CR in the d-wave junctions increases with increasing Z. This argument can be confirmed by the behavior of Φ_ω, which represents a degree of the proximity effect in DN. In Figs. 2(c) and 2(d), we show Φ_ω as a function of ω, where we choose $x=L/2$, $\varphi=\pi/2$, $T/T_C=0.2$, and $(\alpha,\beta)=(0,0)$. The magnitudes of Re Φ_ω and Im Φ_ω increase with increasing Z. This is the reason for the enhancement of I_CR product for low transparent junctions with large magnitude of Z.

Next we focus on the Josephson effect with nonzero values of α and β. Figure 3(a) displays I_CR as a function of temperature for $Z=10$, $R_d/R_b=1$, and $E_{Th}/\Delta(0)=1$. In contrast to I_CR for $(\alpha,\beta)=(0,0)$, the results for $(\alpha,\beta)=(\pi/8,0)$ and $(\pi/8,\pi/8)$ show a nonmonotonic temperature dependence. The transparency at the interfaces greatly changes the peak structure as shown in Fig. 3(b), where temperature dependence of I_CR is plotted for several Z at $(\alpha,\beta)=(\pi/8,0)$. The peak structure is smeared as the decrease of Z.

To understand the origin of the nonmonotonic temperature dependence of I_CR, we have to consider in detail the mechanism of competition between the proximity effect and the formation of the MARS. For example, at $\alpha=\beta=\pi/8$, the injection angles of a quasiparticle are separated into four regions: (i) $\phi_M=\{\phi|\pi/8\leqslant\phi<3\pi/8\}$, (ii) $\phi_P=\{\phi|0\leqslant\phi<\pi/8$ and $3\pi/8\leqslant\phi\leqslant\pi/2\}$, (iii) $-\phi_M$, and (iv) $-\phi_P$. Transport channels within $\pm\phi_P$ contribute to the proximity effect but do not form the MARS. On the other hand, transport channels within $\pm\phi_M$ are responsible for the formation of the MARS but do not contribute to the proximity effect. Thus channels within $\pm\phi_P$ and those within $\pm\phi_M$ play different roles in electron transport. At the same time, the types of wave transmission are different for these two transport channels. The channels within $\pm\phi_P$ are open for the normal transmission. On the other hand, in the channels within $\pm\phi_M$, the resonant transmission is also possible for small ω. At high temperatures, effects of the MARS are negligible and Josephson currents are carried through channels within $\pm\phi_P$. At low temperatures, the resonant transmission through $\pm\phi_M$ governs electric currents. This is a result of an important property of resonant transport. When a normal transport channel and a resonant transport channel are available in parallel, a quasiparticle tends to choose the resonant channel for its transmission at low temperatures. This property can be confirmed in the present junctions by ω dependence of Φ as shown in Figs. 3(c) and 3(d). The results show that Φ rapidly decreases to zero in the limit of $\omega=0$, which implies no diffusion of Cooper pairs into DN through the normal channels within $\pm\phi_P$. At $\omega=0$, quasiparticles choose the resonant channels within $\pm\phi_M$. Electric currents through $\pm\phi_M$, however, do not contribute to the net Josephson current because of cancellation of the currents through ϕ_M by those through $-\phi_M$. Thus Φ goes to zero at $\omega=0$, which leads to the suppression of the Josephson current at low temperatures and hence the nonmonotonic temperature dependence of I_C. The influence of the MARS is more remarkable for larger Z. Therefore the resulting nonmonotonic temperature dependence becomes much more pronounced for large Z as shown in Fig. 3(b). The origin of the nonmonotonic temperature dependence of I_CR in D/I/DN/I/D junctions is different from that in the mirror-type DID junctions (i.e., $\alpha=-\beta$), where the current through the resonant channels causes the 0-π transition and the nonmonotonic temperature dependence.[8,10,11]

It is important to note that the angle average in Eq. (5) at the two interfaces can be carried out independently. The physical meaning can be explained by considering the motion of a quasiparticle which starts from the left interface with a certain injection angle and reaches the right interface after traveling across the DN. At the right interface, injection angles of a quasiparticle are independent of the initial injection angle at the left interface because of the diffusive scatterings in the DN. This property allows one to carry out the angular averaging at the two interfaces independently. When a DN is replaced by a clean normal metal or a clean insulator, the injection angles at both interfaces are correlated to each other. This is because the injection angle of a quasiparticle at the left interface is conserved when the quasiparticle reaches the right interface. As a result, in the clean limit, Josephson current in a symmetric junction (i.e., $\alpha=\beta$) has a behavior different from that in a mirror-type junction with $\alpha=-\beta$. In a symmetric DID junction, I_CR increases monotonically with decreasing temperature.[8,9] In the D/I/DN/I/D junctions considered in the present paper, the mirror-type and the symmetric configurations yield the same Josephson current because of the independent angle averaging at the two interfaces.

To study the nonmonotonic temperature dependence of I_CR further, we define the temperature T_p at which I_CR has a

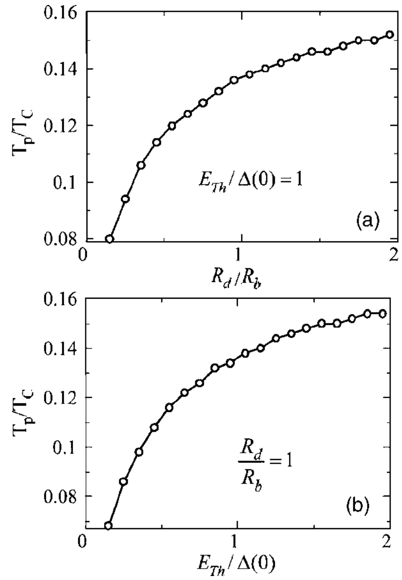

FIG. 4. The peak temperature as a function of R_d/R_b (a) and $E_{Th}/\Delta(0)$ (b) for $Z=10$ and $(\alpha,\beta)=(\pi/8,0)$.

maximum. We have confirmed that T_p is insensitive to Z when Z is sufficiently large. The calculated T_p shown in Fig. 4 increases monotonically with the increase of R_d/R_b and $E_{Th}/\Delta(0)$. These results imply that large magnitudes of Z, R_d/R_b, and $E_{Th}/\Delta(0)$ are needed to observe the peak structure experimentally. These conditions are satisfied in junctions with high insulating barriers, a small cross-section area of the DN, and thin DN.

In the present paper, we have studied the Josephson effect in d-wave superconductor/insulator/diffusive normal metal/insulator/d-wave superconductor junctions by solving the Usadel equations. We have derived the boundary conditions for the quasiclassical Green's function which make it possible to calculate the Josephson current across the structure. By calculating the Josephson current for various parameters, we have clarified the following points. (1) In contrast to the conventional s-wave junctions, the magnitude of I_CR is enhanced with the increase of the insulating barrier strengths at the interfaces. (2) The I_CR value may exhibit a nonmonotonic temperature dependence due to the competition between the proximity effect and the formation of the MARS. The origin of such a nonmonotonic behavior is completely different from that in the established case of a clean DID junction[8,10,11] where 0-π transition occurs due to the MARS. In order to observe experimentally the predicted nonmonotonic temperature dependence of I_C in D/I/DN/I/D junctions, large magnitudes of parameters Z, R_d/R_b, and $E_{Th}/\Delta(0)$ are required. Note that we can reproduce the main conclusions obtained here within the quasiclassical theory of superconductivity by numerical simulations based on the recursive Green's function method.[19]

It follows from the present approach that similar nonmonotonic temperature dependence is also expected for s-wave superconductor/insulator/diffusive normal metal/ insulator/d-wave superconductor junctions. To the best of our knowledge of Josephson junctions consisting of s- and d-wave superconductors, there is only one report in literature about the observation of nonmonotonic temperature dependence of I_C in YBaCuO/Pb junctions.[20] Though this experiment might be possibly relevant to our results, more experimental data are needed to check whether it is the case.

[1] B. D. Josephson, Phys. Lett. **1**, 251 (1962).
[2] K. K. Likharev, Rev. Mod. Phys. **51**, 101 (1979).
[3] A. A. Golubov, M. Yu. Kupriyanov, and E. Il'ichev, Rev. Mod. Phys. **76**, 411 (2004).
[4] V. V. Ryazanov *et al.*, Phys. Rev. Lett. **86**, 2427 (2001).
[5] V. Ambegaokar and A. Baratoff, Phys. Rev. Lett. **10**, 486 (1963).
[6] M. Yu. Kupriyanov and V. F. Lukichev, Zh. Eksp. Teor. Fiz. **94**, 139 (1988) [Sov. Phys. JETP **67**, 1163 (1988)].
[7] A. V. Zaitsev, Physica C **185-189**, 2539 (1991).
[8] Yu. S. Barash, H. Burkhardt, and D. Rainer, Phys. Rev. Lett. **77**, 4070 (1996); Y. Tanaka and S. Kashiwaya, Phys. Rev. B **53**, R11957 (1996); Y. Tanaka and S. Kashiwaya, *ibid.* **56**, 892 (1997).
[9] A. A. Golubov and M. Yu. Kupriyanov, JETP Lett. **69**, 262 (1999).
[10] E. Il'ichev *et al.*, Phys. Rev. Lett. **86**, 5369 (2001).
[11] G. Testa *et al.*, Phys. Rev. B **71**, 134520 (2005).
[12] L. J. Buchholtz and G. Zwicknagl, Phys. Rev. B **23**, 5788 (1981); J. Hara and K. Nagai, Prog. Theor. Phys. **74**, 1237 (1986); C. Bruder, Phys. Rev. B **41**, 4017 (1990); C. R. Hu, Phys. Rev. Lett. **72**, 1526 (1994).
[13] Y. Tanaka and S. Kashiwaya, Phys. Rev. Lett. **74**, 3451 (1995).
[14] Yu. V. Nazarov, Superlattices Microstruct. **25**, 1221 (1999).
[15] K. D. Usadel, Phys. Rev. Lett. **25**, 507 (1970).
[16] Y. Tanaka, Y. V. Nazarov, and S. Kashiwaya, Phys. Rev. Lett. **90**, 167003 (2003).
[17] Y. Tanaka *et al.*, Phys. Rev. B **69**, 144519 (2004).
[18] Y. Tanaka and S. Kashiwaya, Phys. Rev. B **70**, 012507 (2004); Y. Tanaka, S. Kashiwaya, and T. Yokoyama, *ibid.* **71**, 094513 (2005); Y. Tanaka, Y. Asano, A. A. Golubov, and S. Kashiwaya, *ibid.* **72**, 140503(R) (2005).
[19] Y. Asano, Phys. Rev. B **64**, 014511 (2001); J. Phys. Soc. Jpn. **71**, 905 (2002).
[20] I. Iguchi and Z. Wen, Phys. Rev. B **49**, 12388 (1994).