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Josephson Effect in Junctions of Sr_2RuO_4

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Josephson effect between two Sr_2RuO_4 (SRO) is theoretically studied based on a new formula of Josephson current. We analytically calculate the Josephson current in clean SRO/insulator/SRO junctions and SRO/dirty normal metal/SRO junctions. In SRO, we assume a spin-triplet p wave superconductivity which breaks the chiral symmetry. When the current flows in a direction perpendicular to the c axis of SRO, the zero-energy states formed at the junction interfaces enhance the Josephson current in low temperature regime. We also report the Josephson current parallel to the c axis.

KEYWORDS: unconventional superconductor, Andreev reflection, Josephson effect, zero-energy states
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1. Introduction

The anisotropic superconductivity has been an important topics in condensed matter physics since unconventional superconductivity was discovered in heavy-fermion materials.[1] The anisotropic superconductivity was found in a layered perovskite Sr_2RuO_4 (SRO) in a recent study.[2] These materials are classified as spin-triplet superconductors. It is difficult to determine clearly pair potentials of spin-triplet superconductors because Cooper pairs have both spin and orbital degree of freedom. Candidates for the pair potential in SRO have been discussed in a number of studies.[3–13] At present, a pair potential which breaks the chiral symmetry is one of the possible candidates,[5] where the angular momentum of a Cooper pair around the c axis of SRO can be either positive or negative value.

So far the transport properties in SRO/I/s wave superconductors[14–16] and SRO/s wave superconductor/SRO junctions have been studied,[17,18] where I denotes insulators. In general, there is no Josephson coupling between spin-singlet and spin-triplet superconductors when the spin-flip transmission in insulators is absent. In the experiment,[14] however, Josephson current was observed when the c axis of SRO is parallel to the junction interface. This is because the potential step near the insulators become a source of the spin-orbit coupling.[19] No Josephson current was observed when the c axis is perpendicular to the junction interface, which can be also theoretically explained.[19] The Josephson current between two SRO's was theoretically studied in SRO/I/SRO[20] and SRO/C/SRO[21] by using the quasi-classical Green function method, where C is a constriction. In anisotropic superconductor junctions, zero-energy state (ZES)[22,23] formed at the junction interfaces dominates the Josephson current in low temperature regime.[24–26] In high-T_c superconductor junctions, zero-bias conductance peak which is also a consequence of ZES's was observed in a number of normal-metal/superconductor (NS) junctions.[27,28] The zero-bias conductance peak was also discussed in SRO junctions.[29,30] The low-temperature anomaly in the Josephson current is expected when the c axis of SRO is parallel to the junction interface.

In this paper, we apply a new formula for the Josephson current[31,32] to clean SRO/I/SRO junctions and SRO/N/SRO junctions, where N is the dirty normal metal. In the new formula, the Josephson current is described by the Andreev reflection[33] coefficients at the NS interface. Some of our results agree with those in the previous paper,[20] although the theoretical methods for calculating the Josephson current are different from each other. When the c axis of SRO is parallel to the junction interface, ZES's enhance the Josephson current in low temperatures and the amplitudes of the critical current become larger than those in the s wave junctions. In SRO/N/SRO junctions, the ensemble average of the Josephson current vanishes because of the p wave symmetry in the pair potential when the c axis is perpendicular to the interface. Throughout this paper, we take the units of $k_B = \hbar = 1$, where k_B is the Boltzmann constant.

This paper is organized as follows. In §2, we give explicit expressions of Josephson current in SRO/I/SRO and SRO/N/SRO junctions. In §3, the critical Josephson current in these junctions is shown as a function of temperatures. We summarize this paper in §4.

2. Josephson Current Formula

We describe the pair potential of SRO in a simple form, $\hat{\Delta} = i\boldsymbol{d}(\boldsymbol{k}) \cdot \hat{\boldsymbol{\sigma}}\hat{\sigma}_2$ with $d_1(\boldsymbol{k}) = d_2(\boldsymbol{k}) = 0$ and $d_3(\boldsymbol{k}) = \Delta(\bar{k}_1 + \chi i\bar{k}_2)$, where Δ is the amplitude of the pair potential, $\bar{k}_1$, $\bar{k}_2$ and $\bar{k}_3$ are the wavenumber on the Fermi surface divided by the Fermi wavenumber (k_F) in a, b and c axis of SRO, respectively. The Pauli matrices are denoted by $\hat{\sigma}_j$ for $j = 1, 2$ and 3. The parameter $\chi = \pm 1$ corresponds to two possible values of chirality.[20] When $\chi = 1(-1)$, the angular moment of a Cooper pair around the c axis takes a positive (negative) value as shown in Fig. 1(a).

The Josephson current in spin-triplet superconductor/I/spin-triplet superconductor junctions is calculated based on a formula[31]

$$J = 4e \,\mathrm{Im} \sum_{\boldsymbol{p}} T \sum_{\omega_n} \boldsymbol{\Gamma}(\boldsymbol{p}, L)\boldsymbol{\Gamma}^*(\boldsymbol{p}, R), \tag{1}$$

$$\boldsymbol{\Gamma}(\boldsymbol{p}, j) = \bar{k}K_+\boldsymbol{d}_- \mathrm{e}^{i\varphi_j}/\Xi\big|_j, \tag{2}$$

$$\boldsymbol{d}_\pm = \boldsymbol{d}(\pm k, \boldsymbol{p}), \tag{3}$$

$$K_\pm = \sqrt{\omega_n^2 + |\boldsymbol{d}_\pm|^2} - |\omega_n|, \tag{4}$$

$$\Xi = (H^2 + \bar{k}^2)\boldsymbol{d}_+^* \cdot \boldsymbol{d}_- + H^2K_+K_-, \tag{5}$$

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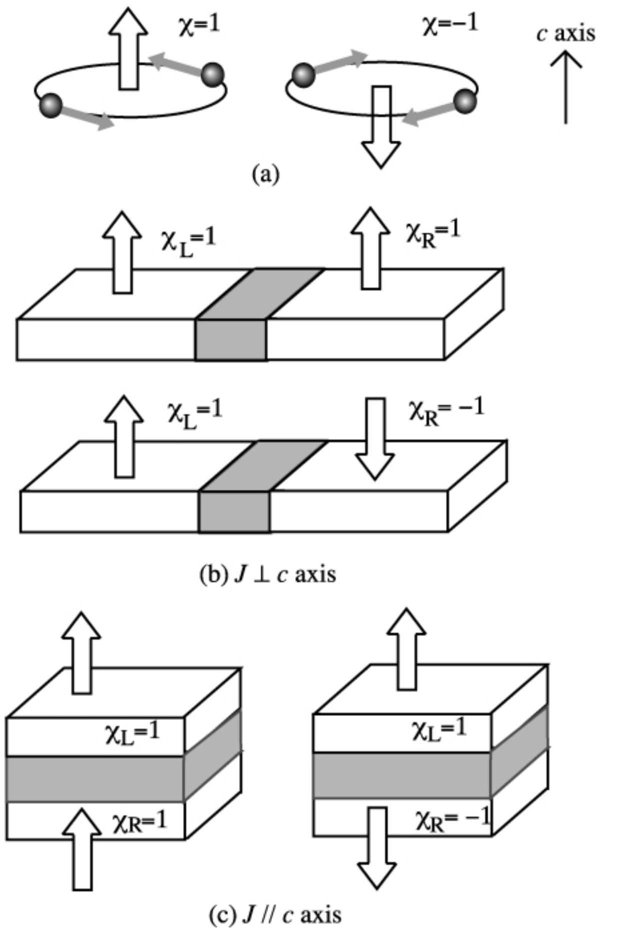

Fig. 1. The two possible states of a Cooper pair are schematically illustrated in (a). The Josephson current is in the direction perpendicular to the c axis in (b). In (c), the Josephson current flows in the parallel direction to the c axis.

where $\omega_n = (2n+1)\pi T$ is the Matsubara frequency, T is a temperature, $H \gg 1$ represents the strength of the barrier potential at insulators and $\varphi_{L(R)}$ is the superconducting phase. The wavenumber on the Fermi surface in the direction normal to the junction interface is $k = \bar{k}k_F$ and that in the parallel plane to the interface is $\boldsymbol{p}$. The vector $\boldsymbol{\Gamma}$ can be connected with the Andreev reflection coefficients, for instance, $\hat{r}^{eh}(\boldsymbol{p}, L) = \boldsymbol{\Gamma}(\boldsymbol{p}, L)\cdot\hat{\boldsymbol{\sigma}}\hat{\sigma}_2$ is the Andreev reflection coefficients at the left junction interface. In these equations, we assume that the spin-triplet superconductors are in unitary states and that the transparency of insulating layers is sufficiently low.[31)]

We also calculate the Josephson current in spin-triplet superconductor/N/spin-triplet superconductor junctions, where N is the normal metal in the diffusive transport regime owing to the impurity scatterings. In analytical calculation, we can obtain the ensemble average of the Josephson current with respect to random impurity configurations,

$$\langle J\rangle = 4eT\sum_{\omega_n} g_N \frac{ln}{\sinh ln}\,\mathrm{Im}\big[\boldsymbol{I}_L\cdot\boldsymbol{I}_R^*\big], \tag{6}$$

$$\boldsymbol{I}_j = \frac{1}{N_c}\sum_{\boldsymbol{p}}\boldsymbol{\Gamma}(\boldsymbol{p}, j), \tag{7}$$

where $\langle\cdots\rangle$ means the ensemble average, $ln = \sqrt{2n+1}L/\xi_D$, $\xi_D = \sqrt{D_0/2\pi T}$ is the coherence length, D_0 is the diffusion constant, L is the length of normal metals in the current direction, N_c is the number of propagation channels on the Fermi surface, and $G_N \equiv (2e^2/h)g_N$ is the ensemble average of the conductance in normal metals. The derivation of the Josephson current formula was discussed in previous papers.[31,34)] We assume the potential barriers at the two NS interfaces, which reflects differences in electronic structures between anisotropic superconductors and simple normal metals. Equation (7) corresponds to the Andreev reflection coefficients averaged over the incident angle of a quasiparticle into a junction interface.

In general, the amplitude of the pair potential near insulators or normal metals deviates from its bulk value and should be determined in a self-consistent way. In the previous paper, the Josephson current calculated with a self-consistent pair potentials was compared with those in a non-self-consistent treatment.[20)] The authors did not find any essential differences between the two results. Thus we do not estimate the pair potential in a self-consistent way in this paper.

3. Results

Firstly we consider SRO/I/SRO junctions where the current flows in a direction perpendicular to the c axis as shown in Fig. 1(b). We neglect the motion of a quasiparticle in the c axis because of the large anisotropy in the electric conduction of SRO.[2)] When the a axis is oriented by $\alpha_{L(R)}$ from the interface normal, a phase $-\chi_j\alpha_j$ is added to φ_j for $j = $ L or R. We should consider that $\tilde{\varphi}_j = \varphi_j - \chi_j\alpha_j$ is a phase of the pair potential because it may be impossible to distinguish the phase of superconductivity φ_j from that stemming from the orientation angle $\chi_j\alpha_j$ in experiments. If it is possible to control φ_j and $\chi_j\alpha_j$ separately in experiments, the phase current relationship deviates from a conventional relation described by $J \propto \sin(\varphi_L - \varphi_R)$.[21)] The vector $\boldsymbol{\Gamma}$ in eq. (2) in this case is given by

$$\boldsymbol{\Gamma}(\gamma, j) = \frac{\Delta K\cos^2\gamma\, e^{i\tilde{\varphi}_j}e^{-i\chi_j\gamma}}{\Delta^2(H^2+\cos^2\gamma)e^{-2i\chi_j\gamma} - H^2K^2}\,\boldsymbol{e}_3, \tag{8}$$

$$K = \sqrt{\omega_n^2 + \Delta^2} - |\omega_n|, \tag{9}$$

where γ represents the incident angle of a quasiparticle measured from the interface normal of two-dimensional junctions and $\bar{k}_1 = \cos\gamma$. When $\chi_L = \chi_R$, the phase-current relationship becomes $J = J_c\sin(\tilde{\varphi}_L - \tilde{\varphi}_R)$, where J_c is the critical Josephson current. In low temperature regime ($T/T_c \ll 1$), J_c results in

$$J_c \approx J_0 1.9\ln\left[\frac{\tau+\zeta}{\zeta(\tau+\zeta)+\tau^2/2}\right], \tag{10}$$

$$J_0 = \frac{\pi\Delta_0}{2eR_J}, \tag{11}$$

$$R_J^{-1} = \frac{16}{15}\frac{e^2}{h}\frac{N_c}{H^4}, \tag{12}$$

where Δ_0 is the amplitude of the pair potential at $T = 0$, $\tau = \pi T/\Delta_0$, $\zeta = (2H^2)^{-1}$, R_J is the normal resistance of the junction. The critical current logarithmically increases with decreasing T and converges to $J_0 1.9\ln(1/\zeta)$ at $T = 0$. The logarithmic increase in the critical current was also pointed out in the previous paper by using the quasi-classical Green function method[20] and is owing to the ZES's at both sides of the junction interfaces. For comparison, we also show the critical current in a junction of the high-T_c superconductors, where a axis is oriented by 45° from the interface normal.[24,25] In this case, the pair potential is given by $d_\pm = \pm\Delta\bar{k}_1\bar{k}_2$. In low temperatures, the critical current results in

$$J_c = J_0 \frac{8}{21\pi}\frac{1}{\tau}\psi'\left(\frac{1}{2}+\frac{\zeta}{4\tau}\right), \tag{13}$$

where $\psi'(x)$ is the tri-gamma function. Although it was pointed out that the critical current is proportional to $1/T$ in many studies, J_c has a finite value of $J_0 \frac{32}{21\pi\zeta}$ at $T = 0$ since $\lim_{x\to\infty}\psi(x) \sim 1/x$. In both cases in eqs. (10) and (13), the Josephson current at $T = 0$ is characterized by ζ which is proportional to $1/\sqrt{R_J}$.[24–26] The ZES appears when a condition $d_+ = -d_-$ ($\boldsymbol{d}_+ = -\boldsymbol{d}_-$) is satisfied for spin-singlet (spin-triplet) superconductors. In high-T_c materials, this condition is always satisfied irrespective of the incident angle of a quasiparticle into the junction interface. On the other hand in SRO, the condition is satisfied only when $k_2 = 0$. This is the origin of the logarithmic dependence of the critical current on T.

When $\chi_L = -\chi_R$, we find

$$J_c \sim J_0(2.2 - 0.9\tau^2). \tag{14}$$

In the previous paper,[20] it is pointed out that there is no unusual behavior in the Josephson effect in this case and J_c seems to saturate in the low temperatures. Our results, however, show that critical current is proportional to T^2 in low temperatures, which is also owing to the ZES's weakly bound at the junction interfaces. In addition to this, the amplitude of J_c becomes larger than that of the s wave junctions because of the ZES's. The ZES is a consequence of an interference effect of a quasiparticle. For $\chi_L = \chi_R$, a quasiparticle interferes constructively. However a quasiparticle interferes rather destructively for $\chi_L = -\chi_R$. In Fig. 2(a), we show the critical current as a function of temperature, where the summation in eq. (1) is numerically carried out and the dependence of Δ on T is given by that of BCS theory. In low temperatures, the amplitudes of J_c are larger than that in the s wave junctions for both cases.

Next we consider the Josephson current where the c axis is perpendicular to the interface as shown in Fig. 1(c). Equation (2) is given by

$$\boldsymbol{\Gamma}_j = \frac{\cos^2\theta\sin\theta\Delta}{2H^2\sqrt{\omega_n^2+\Delta^2\sin^2\theta}}\mathrm{e}^{i\varphi_j}\mathrm{e}^{-i\chi_j\phi}\boldsymbol{e}_3, \tag{15}$$

where $\bar{k}_1 = \sin\theta\cos\phi$, $\bar{k}_2 = \sin\theta\sin\phi$ and $\bar{k}_3 = \cos\theta$.

For $\chi_L = \chi_R$, the critical current becomes

$$J_c = J_0\frac{1}{4c_0}\tanh\left(\frac{c_0\Delta_0}{2T}\right), \tag{16}$$

where c_0 is a numerical factor of the order of unity. In Fig. 2(b), we show the numerical results in this case. The Josephson current saturates in very low temperatures as in the s wave SIS junctions and becomes smaller than J_0 at $T = 0$ because there is no ZES at the interface. For $\chi_L = -\chi_R$, the Josephson current vanishes.[20] This is because the rotational symmetry around the c axis still holds in this case and $\chi = \pm 1$ characterize the two chiral states which are orthogonal to each other.[35,36]

Finally we consider ensemble average of Josephson current in SRO/N/SRO junctions. The critical current does not depend on choices of χ_L and χ_R. When the current is perpendicular to the c axis, the Andreev reflection coefficients averaged over the incident angle of a quasiparticle in eq. (7) becomes

$$\boldsymbol{I}_j = \frac{\mathrm{e}^{i\tilde{\varphi}_j}}{2H^2}\left[\left\{1+\left(\frac{\omega_n}{\Delta}\right)^2\right\}\arctan\left(\frac{\Delta}{\omega_n}\right)-\left(\frac{\omega_n}{\Delta}\right)\right]\boldsymbol{e}_3. \tag{17}$$

The ensemble average of the critical current is proportional to a power of T in low temperatures as shown in Fig. 3, where the degree of disorder in normal metals is chosen to be $L/\xi_D(T_c) = 5$ and 10. The ZES's are weakly formed at the two NS interfaces, but the amplitudes of the critical current does not show the anomalous behavior in low

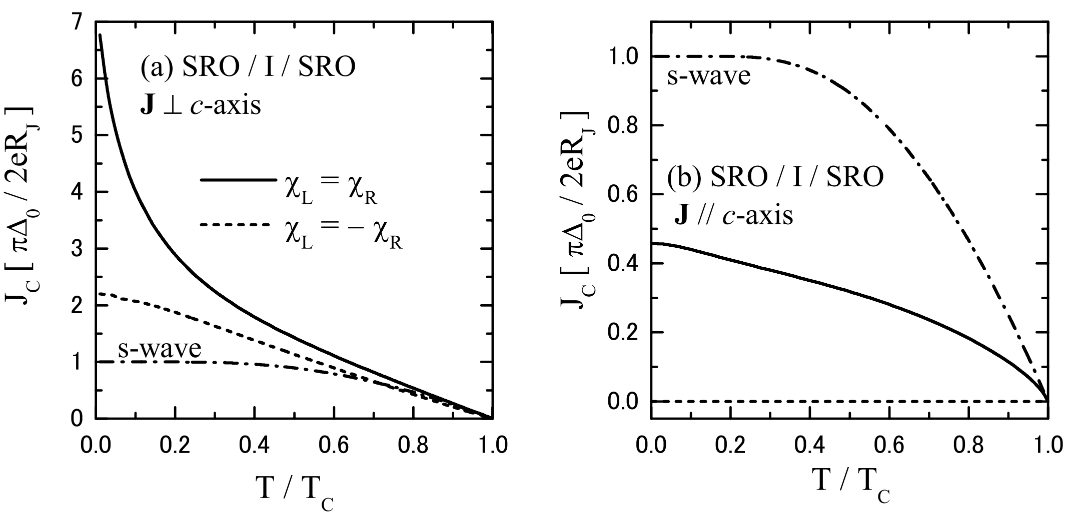

Fig. 2. The critical Josephson current in SRO/I/SRO junctions is plotted as a function of temperatures. In (a), the Josephson current is perpendicular to the c axis of SRO. In (b), the current flows in the direction of the c axis.

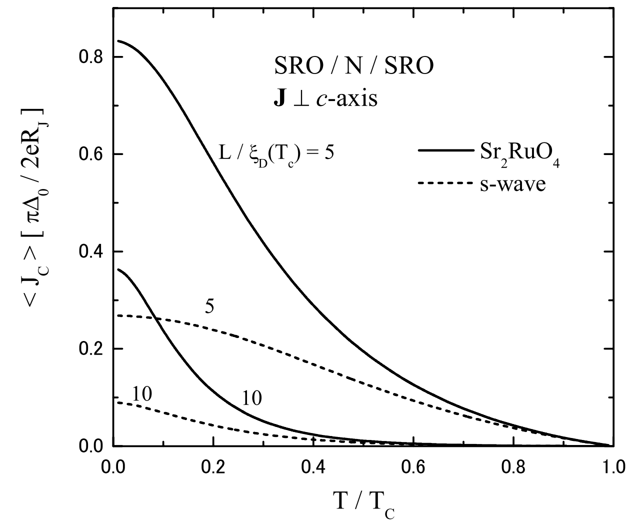

Fig. 3. The critical Josephson current in SRO/dirty *N*/SRO junctions is plotted as a function of temperatures for several choices of degree of disorder in normal metals. The current flows in a direction perpendicular to the *c* axis.

temperatures. For comparison, we show results of *s* wave SNS junctions[37)] with broken lines. The Josephson current in SRO/N/SRO junctions are larger than those in *s* wave junctions in low temperatures because of the ZES's at the NS interfaces. When the current direction is parallel to the *c* axis as shown in Fig. 1(c), the averaged Josephson current vanishes because of the *p* wave symmetry of the pair potential.[38)] In this case, $\boldsymbol{\Gamma}(\boldsymbol{p},j)$ becomes a odd function of $\boldsymbol{p}$, which leads to $\boldsymbol{I}_j = 0$ in eq. (7). We note that $\langle J \rangle = 0$ does not mean an absence of the Josephson current in a single SRO/N/SRO junction with a specific impurity configuration. The Josephson current in a single measurement remains a finite value, but a sign of the Josephson current depends on the random impurity configuration.[34,38,39)] In anisotropic superconductor junctions, it is possible to consider junction where the condition $\boldsymbol{I}_j = 0$ is satisfied.[38)] In these junctions, there is no proximity effect near the junction interface, therefore there is no reflectionless tunneling.[40,41)]

4. Conclusion

The Josephson current between two Sr_2RuO_4 (SRO) has been calculated based on a new formula of Josephson current. We study the Josephson effect in SRO/I/SRO and SRO/N/SRO junctions, where I and N denote insulators and dirty normal metals, respectively. We assume that SRO is a *p* wave spin-triplet superconductor and that the pair potential breaks the chiral symmetry. The zero-energy states enhance the critical current in low temperatures when the Josephson current flows in the direction perpendicular to the *c* axis. In clean SRO/I/SRO junctions, the Josephson current shows a logarithmic low-temperature anomaly when the angular momentum of Cooper pairs in two superconductors align in parallel. When they align in anti-parallel, the Josephson critical current is larger than that in *s* wave junctions but there is no anomalous behavior in low temperatures. In SRO/N/SRO junctions, the Josephson current is independent of alignments of the angular momentum in two SRO's. We show the enhancement of the critical current perpendicular to the *c* axis because of the zero-energy states. The ensemble average of the critical current vanishes when the current is parallel to the *c* axis.

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