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Unconventional Superconductivity and Josephson Effect in Superconductor/Dirty Normal Metal/Superconductor Junctions

Yasuhiro ASANO*

Department of Applied Physics, Hokkaido University, Sapporo 060-8628

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A relation between amplitudes of Josephson current and nodes in excitation-gap of unconventional superconductors is studied in superconductor/dirty normal metal/superconductor junctions. It is found that ensemble average of the Josephson current vanishes when the junction interface is perpendicular to a plane which includes line- or point-zeros of the excitation-gap. A zero-energy state formed at the junction interface is a character of unconventional superconductor junctions and causes large sample-to-sample fluctuations in the Josephson current in low temperatures. The disappearance of the ensemble-averaged Josephson current and the large fluctuations occur at the same time when the Cooper pairs have d- or f-wave pairing symmetries.

KEYWORDS: unconventional superconductor, Andreev reflection, Josephson effect, zero-energy states
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1. Introduction

Symmetries of the Cooper pair are an important information to understand the mechanism of high-T_c superconductivity.[1] Transport properties in anisotropic superconductors have been a topic of increasing interest[2,3] because the high-T_c superconductors may have the $d_{x^2-y^2}$-wave pairing symmetry.[4–6] The unconventional superconductivity has been one of important topics since superconductivity was found in heavy-fermion materials such as, $CeCu_2Si_2$, UBe_{13} and UPt_3.[7–9] The unconventional superconductivity is reported in a layered perovskite Sr_2RuO_4 in a recent work.[10] One of the important features in unconventional superconductor junctions is the formation of zero-energy bound sates (ZES's)[11] at the junction interface. It is known that the ZES's cause a low-temperature anomaly of the Josephson current in SIS junctions of d-wave superconductors, where I denotes insulators.[12,13]

In previous papers, we studied the dc Josephson effect in superconductor/normal metal/superconductor (SNS) junctions of the $d_{x^2-y^2}$-wave superconductors, where the normal metal is in the diffusive transport regime owing to impurity scatterings.[14,15] We found that ensemble average of the Josephson current, $\langle J\rangle$, vanishes when an orientation angle between the a-axis of high-T_c superconductors and the junction interface normal is $\pi/4$. The disappearance of $\langle J\rangle$ is a consequence of the d-wave symmetry (anisotropy) in the pair potential; a sign of the pair potential becomes either positive or negative along a Fermi surface. Thus $\langle J\rangle = 0$ is expected in dirty SNS junctions of anisotropic superconductors with another symmetries.

In this paper, we show that the ensemble average of Josephson current vanishes when the junction interface is perpendicular to a plane which includes the line- and the point-zeros of an excitation-gap irrespective of symmetries in the pair potentials. The work reported here is a natural extension of the previous work.[15] Throughout this paper, we take the units of $\hbar = k_B = 1$, where k_B is the Boltzmann constant.

This paper is organized as follows. In §2, we briefly explain a formula for the Josephson current used in calculation. The Josephson current in dirty SNS junctions of p-, d-, and f-wave superconductors is studied in §3. In §4, we discuss a meaning of the disappearance of the ensemble-averaged Josephson current. The conclusion is given in §5.

2. Josephson Current

Let us consider an SNS junction, where the length of the normal metal is L_N and the cross section of the junction is S_J. The normal of the junction interface is in the z direction. In the xy plane, the periodic boundary condition is applied. At the two NS interface, ($z = 0$ and L_N), the potential barrier described by $V_b\{\delta(z) + \delta(z - L_N)\}$ is introduced, which reflects difference of electronic structures in the normal metal and those in the unconventional superconductors. We describe the SNS junction by the Bogoliubov-de Gennes (BdG) equation,[17]

$$\int d\boldsymbol{r}' \begin{bmatrix} \delta(\boldsymbol{r}-\boldsymbol{r}')h_0(\boldsymbol{r}')\hat{\sigma}_0 & \hat{\Delta}(\boldsymbol{r}-\boldsymbol{r}') \\ -\hat{\Delta}^*(\boldsymbol{r}-\boldsymbol{r}') & -\delta(\boldsymbol{r}-\boldsymbol{r}')h_0(\boldsymbol{r}')\hat{\sigma}_0 \end{bmatrix} \begin{bmatrix} \hat{u}_\lambda(\boldsymbol{r}') \\ \hat{v}_\lambda(\boldsymbol{r}') \end{bmatrix} = \begin{bmatrix} \hat{u}_\lambda(\boldsymbol{r}) \\ \hat{v}_\lambda(\boldsymbol{r}) \end{bmatrix} \hat{E}_\lambda, \tag{1}$$

$$\hat{E}_\lambda = \begin{pmatrix} E_{\lambda,1} & 0 \\ 0 & E_{\lambda,2} \end{pmatrix}, \tag{2}$$

where $h_0(\boldsymbol{r}) = -\frac{\nabla^2}{2m} + V(\boldsymbol{r}) - \mu_F$, μ_F is the Fermi energy, $\hat{\sigma}_0$ is the unit matrix of 2×2. The potential $V(\boldsymbol{r})$ includes the barrier potential at the two NS interfaces and impurity potentials in the normal metal. The pair potential between an electron with (σ, $\boldsymbol{r}$) and that with (σ', $\boldsymbol{r}'$) is given by $\Delta_{\sigma,\sigma'}(\boldsymbol{r}-\boldsymbol{r}')$. In the normal segment, the pair potential is taken to be zero. In what follows, we use $\widehat{\cdots}$ for indicating 2×2 matrices which represents the spin space. Eigenvalues of the BdG equation depend on the spin configuration when the superconductors are in nonunitary states as shown in $E_{\lambda,l}$, where $l = 1, 2$ indicate the spin configuration. In unitary states, $E_{\lambda,1} = E_{\lambda,2}$. In the superconductors, we assume that the scalar and the pair potentials are uniform. Thus the BdG equation is represented in Fourier space,

*E-mail: asano@eng.hokudai.ac.jp

$$\begin{bmatrix} \xi_k \hat{\sigma}_0 & \hat{\Delta}(\boldsymbol{k}) \\ -\hat{\Delta}^*(-\boldsymbol{k}) & -\xi_k \hat{\sigma}_0 \end{bmatrix} \begin{bmatrix} \hat{u}_{\boldsymbol{k}} \\ \hat{v}_{\boldsymbol{k}} \end{bmatrix} = \begin{bmatrix} \hat{u}_{\boldsymbol{k}} \\ \hat{v}_{\boldsymbol{k}} \end{bmatrix} \hat{E}_{\boldsymbol{k}}, \tag{3}$$

where $\xi_k = \boldsymbol{k}^2/(2m) - \mu_{\mathrm{F}}$. The pair potential is given by

$$\hat{\Delta}(\boldsymbol{r} - \boldsymbol{r}') = \sum_{\boldsymbol{k}} \hat{\Delta}(\boldsymbol{k}) \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')}, \tag{4}$$

$$\hat{\Delta}(\boldsymbol{k}) = \begin{cases} i d_0(\boldsymbol{k}) \hat{\sigma}_2 & \text{singlet} \\ i(\boldsymbol{d}(\boldsymbol{k}) \cdot \hat{\boldsymbol{\sigma}}) \hat{\sigma}_2 & \text{triplet} \end{cases}, \tag{5}$$

where $\hat{\sigma}_j$ with $j = 1, 2$ and 3 are the Pauli's matrices. In what follows, we assume that the two superconductors are identical to each other. When the two superconductors have spin-singlet Copper pairs, the Josephson current is given by,[16)]

$$J = 4e \sin\varphi T \sum_{\omega_n} \sum_{\boldsymbol{p},\boldsymbol{p}'} \Gamma_s(\boldsymbol{p}, L) \Gamma_s(\boldsymbol{p}', R) T_N(\boldsymbol{p}, \boldsymbol{p}'), \tag{6}$$

where $\varphi = \varphi_L - \varphi_R$ is the phase difference between the two superconductors, $\boldsymbol{p} = (k_x, k_y)$ is the wavevector in the xy plane, T is a temperature and $\omega_n = (2n+1)\pi T$ is the Matsubara frequency.[16)] The wavevector $\boldsymbol{p}$ ($\boldsymbol{p}'$) indicates the propagating channel at the left (right) NS interface and $T_N(\boldsymbol{p}, \boldsymbol{p}')$ is the transmission coefficients in normal metal between the channel $\boldsymbol{p}$ and $\boldsymbol{p}'$. A part of the Andreev reflection[18)] coefficient (ARC) at left (right) interface is described by $\Gamma_s(\boldsymbol{p}, L)$ ($\Gamma_s(\boldsymbol{p}', R)$). These are a function of

$$d_0 \equiv d_0(\boldsymbol{p}, k_z) \quad \text{and} \quad d_{0,-} \equiv d_0(\boldsymbol{p}, -k_z). \tag{7}$$

It is possible to obtain a general expression of the ARC's.[16)] For purposes of this paper, we need only the ARC under a condition

$$d_0 = \nu d_{0,-}, \tag{8}$$

where $\nu = 1$ or -1. When eq. (8) is satisfied, we obtain

$$\Gamma_s(\boldsymbol{p}, j) = \bar{k}_z^2 \left.\frac{d_0}{\Xi_s}\right|_j, \tag{9}$$

$$\Xi_s = H^2\{(1+\nu)\Omega + (1-\nu)|\omega_n|\} + \bar{k}_z^2(\Omega + |\omega_n|), \tag{10}$$

$$\Omega = \sqrt{\omega_n^2 + |d_0|^2}, \tag{11}$$

for $j = L$ or R, where $H = V_b m / k_F$ represents a strength of the potential barrier at the NS interfaces. When the two superconductors have spin-triplet Copper pairs, the Josephson current is given by,

$$J = 4eT \sum_{\omega_n} \sum_{\boldsymbol{p},\boldsymbol{p}'} \mathrm{Im}[\mathrm{e}^{\mathrm{i}\varphi} \boldsymbol{\Gamma}_t(\boldsymbol{p}, L) \cdot \boldsymbol{\Gamma}_t^*(\boldsymbol{p}', R) T_N(\boldsymbol{p}, \boldsymbol{p}')], \tag{12}$$

where $\boldsymbol{\Gamma}_t(\boldsymbol{p}, L)$ ($\boldsymbol{\Gamma}_t(\boldsymbol{p}', R)$) is a vector stemming from the ARC at the left (right) NS interface[16)] and is a function of

$$\boldsymbol{d} \equiv \boldsymbol{d}(\boldsymbol{p}, k_z) \quad \text{and} \quad \boldsymbol{d}_- \equiv \boldsymbol{d}(\boldsymbol{p}, -k_z). \tag{13}$$

Under the condition

$$\boldsymbol{d} = \nu \boldsymbol{d}_-, \tag{14}$$

with $\nu = 1$ or -1, these vectors are calculated to be

$$\boldsymbol{\Gamma}_t(\boldsymbol{p}, j) = \bar{k}_z^2 \left.\frac{\boldsymbol{d}}{\Xi_u}\right|_j, \tag{15}$$

$$\Xi_u = H^2\{(1+\nu)\Omega + (1-\nu)|\omega_n|\} + \bar{k}_z^2(\Omega + |\omega_n|), \tag{16}$$

$$\Omega = \sqrt{\omega_n^2 + |\boldsymbol{d}|^2}, \tag{17}$$

for unitary states, and

$$\boldsymbol{\Gamma}_t(\boldsymbol{p}, j) = \bar{k}_z^2 \frac{1}{2} \sum_{l=1}^{2} \frac{\boldsymbol{D}_l^*}{\Xi_{nu}(l)}, \tag{18}$$

$$\boldsymbol{D}_{1(2)} = \boldsymbol{d}^* + (-) i \frac{\boldsymbol{d}^* \times \boldsymbol{q}}{|\boldsymbol{q}|}, \tag{19}$$

$$\boldsymbol{q} = i \boldsymbol{d} \times \boldsymbol{d}^*, \tag{20}$$

$$\Xi_{nu}(l) = H^2\{(1+\nu)\Omega_l + (1-\nu)|\omega_n|\} + \bar{k}_z^2(\Omega_l + |\omega_n|), \tag{21}$$

$$\Omega_l = \sqrt{\omega_n^2 + |\Delta_l|^2}, \tag{22}$$

$$|\Delta_{1(2)}| = \sqrt{|\boldsymbol{d}|^2 + (-)|\boldsymbol{q}|}, \tag{23}$$

for nonunitary states.

3. Dirty SNS Junctions

In what follows, we consider superconductors in which the excitation-gap of a quasiparticle has point- and/or line-nodes. We define that 'node-vector' points the direction parallel to a line which connects two node-points as shown in Fig. 1(a), where we illustrate the excitation-gap in the Anderson–Brinkman–Morel states[19)] described by $\hat{\Delta}(\boldsymbol{k}) =$

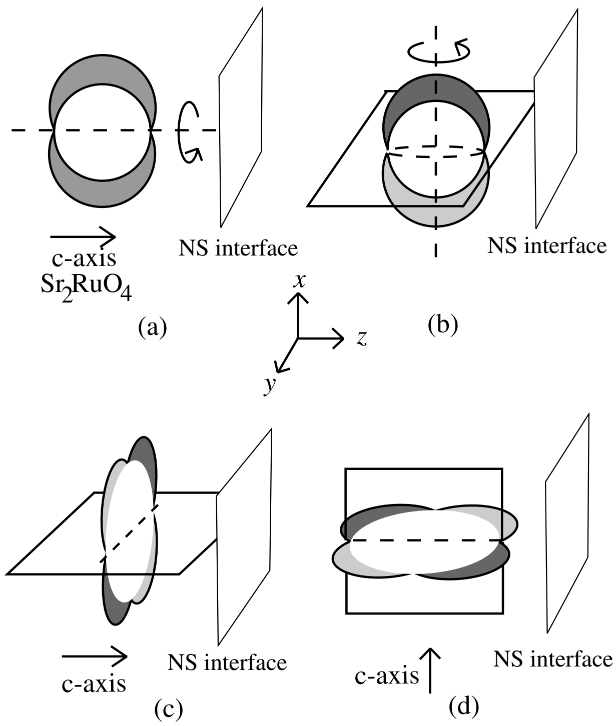

Fig. 1. We schematically illustrate the excitation gap of a quasiparticle in the ABM state (a) and that in the polar state (b) of the p-wave superconductor. In (a), the 'node-axis' corresponds to the axial vector which characterizes the pair potential and is in the z direction in this figure. The 'node-plane' includes the node-lines of the gap and is the yz plane in (b). In (c) and (d), the pair potentials in high-T_C superconductors are shown. The c-axis is perpendicular to the NS interface in (c). In (d), the c-axis is parallel to the NS interface and a node-plane (xz plane) is perpendicular to the NS interface.

$\bar{\Delta}(\bar{k}_x + i\bar{k}_y)\hat{\sigma}_3$. Here $\bar{\Delta}$ is the amplitude of the pair potential and $\bar{k}_j = k_j/k_F$ for $j = x, y$, and z, respectively. Since the profile of the gap in the xz plane is shown in (a), the figure is rotated around the broken line to obtain the real excitation-gap. In the figure, dark areas indicate the gap. The excitation-gap becomes zero at the two points on the Fermi surface (i.e., $\bar{k}_z = \pm 1$). Thus the 'node-vector' in this case is in the z direction. In the same way, we define that 'node-plane' is the plane on which node-lines are included as shown in Fig. 1(b), where we depict the excitation-gap in a polar state described by $\hat{\Delta}(\boldsymbol{k}) = \bar{\Delta}\bar{k}_x\hat{\sigma}_1$. The gap has line-zeros on the equator. Thus a 'node-plane' in this case is the yz plane.

When the normal conductor is in the diffusive transport regime, $T_N(\boldsymbol{p}, \boldsymbol{p}')$ is almost independent of the propagating channels.[15] Therefore we approximately replace these transmission coefficients in the normal metal by its ensemble average

$$\langle T_N(\boldsymbol{p}, \boldsymbol{p}')\rangle = \left(\frac{1}{N_c}\right)^2 g_N \frac{ln}{\sinh ln}, \tag{24}$$

where $ln = \sqrt{2n+1}L_N/\xi_D$, $\xi_D = \sqrt{D_0/2\pi T}$ is the coherence length, D_0 is the diffusion constant, $N_c = S_J k_F^2/4\pi$ is the number of propagation channels on the Fermi surface, $G_N \equiv (2e^2/h)g_N$ is the conductance in the normal metal and $\langle\cdots\rangle$ means the ensemble average with respect to the random configuration of the impurities.[15] Since the transmission coefficients in normal metal are independent of the propagating channels, the summation with respect to $\boldsymbol{p}'$ and $\boldsymbol{p}$ in eqs. (6) and (12) can be carried out independently at the two NS interfaces. The ensemble average of the Josephson current in dirty SNS junctions is given by

$$\langle J\rangle = 4e\sin\varphi T\sum_{\omega_n} g_N \frac{ln}{\sinh ln} I_L I_R, \tag{25}$$

$$I_j = \frac{1}{N_c}\sum_{\boldsymbol{p}} \Gamma_s(\boldsymbol{p}, j), \tag{26}$$

for spin-singlet superconductor junctions, and

$$\langle J\rangle = 4eT\sum_{\omega_n} g_N \frac{ln}{\sinh ln} \mathrm{Im}[e^{i\varphi}\boldsymbol{I}_L\cdot\boldsymbol{I}_R^*], \tag{27}$$

$$\boldsymbol{I}_j = \frac{1}{N_c}\sum_{\boldsymbol{p}} \boldsymbol{\Gamma}_t(\boldsymbol{p}, j), \tag{28}$$

for spin-triplet superconductor junctions.

3.1 *p-wave superconductors*

Although a large number of studies have made on the spin-triplet superconductors, there is not perfect agreement as to the pair potential in many superconductors. Here we study a relation between the nodes in the excitation-gap and the amplitude of the Josephson current without explicit expression of the pair potential. In general, the pair potential of the p-wave superconductor can be described by,

$$\boldsymbol{d}(\boldsymbol{k}) = \bar{\Delta}\sum_{j=1}^{3} \boldsymbol{e}_j[c_{1,j}\bar{k}_x + c_{2,j}\bar{k}_y + c_{3,j}\bar{k}_z], \tag{29}$$

where $c_{1,j}$, $c_{2,j}$ and $c_{3,j}$ are numerical coefficients, $\boldsymbol{e}_1$, $\boldsymbol{e}_2$ and $\boldsymbol{e}_3$ are the unit vector in the x, y and z directions, respectively. When the gap of the p-wave superconductors has the two point-nodes on the k_z-axis, the pair potential is given by

$$\boldsymbol{d}(\boldsymbol{k}) = \bar{\Delta}\sum_{j=1}^{3} \boldsymbol{e}_j[c_{1,j}\bar{k}_x + c_{2,j}\bar{k}_y]. \tag{30}$$

When the gap has the line-node on a plane characterized by $ax + by = 0$, the pair potential is given by

$$\boldsymbol{d}(\boldsymbol{k}) = \bar{\Delta}\sum_{j=1}^{3} c_{1,j}\boldsymbol{e}_j(a\bar{k}_x + b\bar{k}_y). \tag{31}$$

The node-vector in eq. (30) and the node-plane in eq. (31) are perpendicular to the NS interface. In both cases in eqs. (30) and (31), we find $\boldsymbol{d}_- = \boldsymbol{d}$ (i.e., $\nu = 1$). Thus there are no ZES at the NS interfaces. Since Ξ_u in eq. (15) and $\Xi_{nu}(l)$ in eq. (18) are functions of $|\boldsymbol{d}|$ and $|\boldsymbol{q}|$, they are the even function of $\boldsymbol{p}$. On the other hand, $\boldsymbol{d}$ in eq. (15) and $\boldsymbol{D}_l$ in eq. (18) are the odd function of $\boldsymbol{p}$. This fact immediately leads to the disappearance of the averaged Josephson current because the integration with respect to $\boldsymbol{p}$ in eqs. (28) gives zero. Thus we conclude that the averaged Josephson current vanishes when the node-plane or the node-vector of the excitation-gap are perpendicular to the NS interface.

We apply above argument to realistic junctions of the p-wave superconductor Sr_2RuO_4.[10] There is fairly general agreement that the pair potential may be described by[20]

$$\boldsymbol{d} = \bar{\Delta}(\bar{k}_x + i\bar{k}_y)\boldsymbol{e}_3. \tag{32}$$

As well as high-T_c superconductors, the electric conductivity in two-dimensional ab-plane is much larger than that along the c-axis. We consider the situation where c-axis is perpendicular to the NS interface as shown in Fig. 1(a). The superconductor is in the unitary states and the node-vector is in the z direction perpendicular to the NS interface. The integration with respect to $\boldsymbol{p}$ in eq. (28) is given in the limit of $H \gg 1$

$$\boldsymbol{I}_L = \frac{1}{4\pi}\int_0^{2\pi} d\phi e^{i\phi}\int_0^{\pi/2} d\theta \frac{\sin^4\theta\cos^3\theta K}{\Xi_u}\boldsymbol{e}_3, \tag{33}$$

$$\Xi_u = H^2(\bar{\Delta}^2\sin^2\theta + K^2), \tag{34}$$

$$K = \sqrt{\omega_n^2 + \bar{\Delta}^2\sin^2\theta} - |\omega_n|, \tag{35}$$

where use the polar coordinates. It is easily confirmed that the averaged Josephson current vanishes because the integration with respect to ϕ gives zero.

3.2 *d-wave superconductors*

It is known that the pair potential in high-T_c superconductors have $d_{x^2-y^2}$-wave pairing. Thus we focus on the Josephson effect between the two $d_{x^2-y^2}$-wave superconductors. First we consider the situation where the c-axis of the high-T_c superconductor is perpendicular to the NS interface. The pair potential can be described by

$$d_0 = d_{0,-} = \bar{\Delta}(\bar{k}_x^2 - \bar{k}_y^2). \tag{36}$$

We illustrate excitation gap on the ab-plane in Fig. 1(c). The node-planes are described by $x = \pm y$ which are perpendicular to the NS interface. In the limit of high potential barrier at the NS interface (i.e., $H \gg 1$), eq. (26) becomes

$$I_L = \frac{\bar{\Delta}}{2\pi H^2}\int d\bar{k}_x \int d\bar{k}_y \frac{\bar{k}_z^2(\bar{k}_x^2 - \bar{k}_y^2)}{\sqrt{\omega_n^2 + \bar{\Delta}^2(\bar{k}_x^2 - \bar{k}_y^2)^2}}, \tag{37}$$

$$= \frac{\bar{\Delta}}{2\pi H^2}\int_0^{2\pi} d\phi \int_0^{\pi/2} d\theta \frac{\sin^3\theta\cos^2\theta\cos 2\phi}{\sqrt{\omega_n^2 + \bar{\Delta}^2\sin^4\theta\cos^2 2\phi}}. \tag{38}$$

It is shown that the average of the Josephson current vanishes because the integration with respect to ϕ gives zero in eq. (38).

Next we consider the situation where the c-axis is parallel to x direction and the node-plane is perpendicular to the NS interface. After appropriate rotation of the coordinates, the pair potential is described by

$$d_0 = -d_{0,-} = 2\bar{\Delta}\bar{k}_y\bar{k}_z \tag{39}$$

The excitation gap is shown in Fig. 1(d), where one of the node-plane described by $y = 0$ is perpendicular to the NS interface. In this case, Ξ_s is the even function of $\bar{k}_y$, whereas d_0 is the odd function of $\bar{k}_y$. By using eq. (39), eq. (26) becomes

$$I_L = \frac{\bar{\Delta}}{\pi H^2}\int d\bar{k}_x \int d\bar{k}_y \frac{\bar{k}_z^3\bar{k}_y}{\left[|\omega_n| + \frac{4\bar{\Delta}^2\bar{k}_z^4\bar{k}_y^2}{KH^2}\right]}. \tag{40}$$

It is easy to confirm $I_L = 0$ because the integration with respect to $\bar{k}_y$ becomes zero. We note that the ZES are formed at the NS interface since the pair potential satisfies $d_0 = -d_{0,-}$ (i.e., $\nu = -1$). The denominator in the integrand of eq. (40) approaches to zero in the limit of $H \gg 1$ and $\omega_n \to 0$, which indicate that the contribution of ZES to the Josephson current is important in low temperature regime. The ZES are the origin of the large sample-to-sample mesoscopic fluctuations[21,22] in the Josephson current for $T \to 0$.[15]

3.3 *f-wave superconductors*

Although UPt_3 is one of the candidate of f-wave superconductors,[23] we lack a perfect agreement of the f-wave superconductivity in real materials. In addition, it seems to be difficult to give a general discussion on the relation between the node and the Josephson current because the number of the basis functions in f-wave case is too many. However, it is possible to repeat the same argument which we have done in the d- and p-wave cases. We approximately separate the pair potential into two groups,

$$\boldsymbol{d}_1(\boldsymbol{k}) = \bar{\Delta}\sum_{j=1}^{3}\boldsymbol{e}_j[c_{1,j}\bar{k}_x\bar{k}_y\bar{k}_z + c_{2,j}\bar{k}_z(\bar{k}_x^2 - \bar{k}_y^2) + c_{3,j}\bar{k}_z(5\bar{k}_z^2 - 1)], \tag{41}$$

and

$$\boldsymbol{d}_2(\boldsymbol{k}) = \bar{\Delta}\sum_{j=1}^{3}\boldsymbol{e}_j[c_{4,j}\bar{k}_x(5\bar{k}_x^2 - 3) + c_{5,j}\bar{k}_y(5\bar{k}_y^2 - 3) + c_{6,j}\bar{k}_x(\bar{k}_y^2 - \bar{k}_z^2) + c_{7,j}\bar{k}_y(\bar{k}_z^2 - \bar{k}_x^2)]. \tag{42}$$

It is noted that we do not classify the pair potential into two groups in terms of the lattice symmetries.[24] Here we use the parity in eq. (14); $\boldsymbol{d}_1(\boldsymbol{k})$ and $\boldsymbol{d}_2(\boldsymbol{k})$ satisfy $\nu = -1$ and 1, respectively. In $\boldsymbol{d}_1$, the functions $\bar{k}_x\bar{k}_y\bar{k}_z$ and $\bar{k}_z(\bar{k}_x^2 - \bar{k}_y^2)$ have the node-plane at $\bar{k}_x = 0$, $\bar{k}_y = 0$ and $\bar{k}_x = \pm\bar{k}_y$ which are perpendicular to the NS interface. Since the node-plane of $\bar{k}_z(5\bar{k}_z^2 - 1)$ is parallel to the NS interface, we set $c_{3,j}$ to be zero in what follows. In the same way in $\boldsymbol{d}_2$, the basis function become zero on the plane $\bar{k}_x = 0$ or $\bar{k}_y = 0$ which are the perpendicular to the NS interface. These pair potentials in eqs. (41) and (42) can be described in the different linear combinations,

$$\boldsymbol{d}_1(\boldsymbol{k}) = \bar{\Delta}\sum_{j=1}^{3}\boldsymbol{e}_j[\tilde{c}_{1,j}\bar{k}_z(\bar{k}_x + i\bar{k}_y) + \tilde{c}_{2,j}\bar{k}_z(\bar{k}_x - i\bar{k}_y)], \tag{43}$$

and

$$\boldsymbol{d}_2(\boldsymbol{k}) = \bar{\Delta}\sum_{j=1}^{3}\cdot\boldsymbol{e}_j[\{\tilde{c}_{6,j}(\bar{k}_x + i\bar{k}_y) + \tilde{c}_{7,j}\bar{k}_y(\bar{k}_x - i\bar{k}_y)\} \times (5\bar{k}_z^2 - 3) + \tilde{c}_{4,j}(\bar{k}_x + i\bar{k}_y)^3 + \tilde{c}_{5,j}(\bar{k}_x - i\bar{k}_y)^3]. \tag{44}$$

In these expression, it is easily confirmed that the excitation-gap has the two point nodes on the k_z axis. As we have done in the p- and d-wave cases, it can be confirmed that the integration with respect to ϕ in eq. (28) gives zero when we substitute eqs. (41), (42), (43) and (44) into eq. (15) or (18). Thus in the f-wave superconductors, $\langle J\rangle$ vanishes when the node-plane and/or node-vector is perpendicular to the NS interface. When the pair potential is described by eq. (41), the ZES dominate the Josephson current in a single sample and the mesoscopic fluctuations in the Josephson current becomes large in the low-temperature regime. On the other hand when the pair potential is described by eq. (42), the Josephson current in a single sample is also expected to be a small value because there are no ZES.

4. Discussion

In this section, we discuss a meaning of the disappearance of the ensemble-averaged Josephson current. Here we focus on a d-wave superconductor junction, where the pair potential is shown in Fig. 1(d). In clean SIS junctions, the Josephson current rapidly increases with decreasing the temperature because ZES are formed at the junction interface.[12,13] The Josephson current is also studied in clean SNS junctions.[25] In these clean junctions, the Josephson current has finite amplitudes below the critical temperature. It is evident that the Josephson current in a single specific dirty SNS junction must be smaller than that in a clean junction because of the impurity scatterings. However, the amplitude of the Josephson current of a single sample does not banish below T_C, whereas its ensemble-average is zero for all temperatures.[15] This fact implies an importance of the sample-to-sample fluctuations of the Josephson current (δJ) which become large in low temperatures because of ZES.[15] The average of the Josephson current corresponds to the expected value for a measurement of a single sample and the fluctuations indicate the reliability of the measured results. When $\langle J\rangle = 0$ and $\delta J \neq 0$ are satisfied at the same time, the reliability of the Josephson current of a single sample becomes questionable. Actually, we have confirmed that the dependence of the Josephson current on temperatures in one sample is very different from that in another samples.[15] Thus the reliability of a single measurement is lost owing to the impurities. This is the most important effect of the impurities. In another numerical simulation,[26] we confirmed $\langle J \rangle = 0$ even when the normal metal is in the quasi-ballistic transport regime. In experiments[27] at present, the two superconductors are separated by the grain boundary. It seems to be difficult to know the potential profile of the grain boundary. In such situation, experimentalists should pay attention to a fact that the reliability of measured results in a single sample may be questionable if the grain boundary is not clean enough.

The dirty SNS junctions in this paper can be realized in an experiment,[28] where the weak link is fabricated between the two superconductors by the ion irradiation. When the length of the weak link is long enough, the system becomes the dirty SNS junctions because the ion irradiation makes the potential in the weak link random. It is possible to confirm $\langle J \rangle = 0$, when the current-phase relation ship is measured[27] over a number of different samples.

5. Conclusion

We study the ensemble average of the Josephson current in superconductor/dirty normal metal/superconductor junctions of unconventional superconductors. We conclude that the ensemble average of the Josephson current vanishes when the node-plane and/or the node-vector in the quasiparticle's excitation-gap is perpendicular to the NS interface.

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1) J. G. Bednorz and K. A. Müller: Z. Phys. B **64** (1986) 189.
2) S. Kashiwaya and Y. Tanaka: Rep. Prog. Phys. **63** (2000) 1641.
3) T. Löfwander, V. S. Shumeiko and G. Wendin: Supercond. Sci. Technol. **14** (2001) R53.
4) M. Sigrist and T. M. Rice: J. Phys. Soc. Jpn. **61** (1992) 4283.
5) M. Sigrist and T. M. Rice: Rev. Mod. Phys. **67** (1995) 503.
6) D. A. Wollman, D. J. van Harlingen, W. C. Lee, D. M. Ginsberg and A. J. Leggett: Phys. Rev. Lett. **71** (1993) 2134.
7) F. Steglich, J. Aarts, C. D. Bredl, W. Lieke, D. Meschede, W. Franz and H. Schäfer: Phys. Rev. Lett. **43** (1979) 1892.
8) H. R. Ott, H. Rudigier, Z. Fisk and J. L. Smith: Phys. Rev. Lett. **50** (1983) 1595.
9) G. R. Stewart, Z. Fisk, J. O. Willis and J. L. Smith: Phys. Rev. Lett. **52** (1984) 679.
10) Y. Maeno, H. Hashimoto, K. Yoshida, S. Nishizaki, T. Fujita, J. G. Bednorz and F. Lichtenberg: Nature **372** (1994) 532.
11) Y. Tanaka and S. Kashiwaya: Phys. Rev. Lett. **74** (1995) 3451.
12) Y. S. Barash, H. Burkhardt and D. Rainer: Phys. Rev. Lett. **77** (1996) 4070.
13) Y. Tanaka and S. Kashiwaya: Phys. Rev. B **53** (1996) R11957.
14) Y. Asano: Phys. Rev. B **63** (2001) 052512.
15) Y. Asano: Phys. Rev. B **64** (2001) 014511.
16) Y. Asano: Phys. Rev. B **64** (2001) 224515.
17) P. G. de Gennes: *Superconductivity of Metals and Alloys* (Benjamin, New York, 1966).
18) A. F. Andreev: Zh. Eksp. Theor. Fiz. **46** (1964) 1823 [Sov. Phys. JETP **19** (1964) 1228].
19) P. W. Anderson and P. Morel: Phys. Rev. **123** (1961) 1911.
20) C. Honerkamp and M. Sigrist: Prog. Theor. Phys. **100** (1998) 53.
21) B. L. Al'tshuler and B. Z. Spivak: Pis'ma Zh. Eksp. Teor. Fiz. **65** (1987) 609 [Sov. Phys. JETP **65** (1987) 343].
22) C. W. J. Bennakker: Phys. Rev. Lett. **67** (1991) 3836.
23) T. Ohmi and K. Machida: Phys. Rev. Lett. **71** (1993) 625.
24) M. Sigrist and K. Ueda: Rev. Mod. Phys. **63** (1991) 239.
25) R. A. Riedel and P. F. Bagwell: Phys. Rev. B **57** (1998) 6084.
26) Y. Asano: Physica C **367** (2002) 157.
27) E. Il'ichev, M. Grajcar, R. Hlubina, R. P. J. IJsselsteijn, H. E. Hoenig, H.-G. Meyer, A. Golubov, M. H. S. Amin, A. M. Zagoskin, A. N. Omelyanchouk and M. Y. Kupriyanov: Phys. Rev. Lett. **86** (2001) 5369.
28) S. S. Tinchev: Physica C **222** (1994) 173.