Tunneling between Two Helical Superconductors via Majorana Edge Channels

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We discuss electric transport through a point contact which bridges Majorana fermion modes appearing at edges of two helical superconductors. The contents focus on the effects of interference and interaction unique to the Majorana fermions and the role of spin-orbit interaction (SOI). Besides the Josephson current, the quasiparticle conductance depends sensitively on the phase difference and relative helicity between the two superconductors. The interaction among the Majorana fermions causes the power-law temperature dependences of conductance for various tunneling channels. Especially, in the presence of SOI, the conductance always increases as the temperature is lowered.

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Fractionalization of electrons attracts recent intensive interest. A chiral fermion at an edge of quantum Hall system is an example of fractionalized electrons. A fully gapped bulk state spatially separates a right-going and left-going chiral fermion, which leads to the absence of the backward scattering [1]. The robustness against disturbances such as disorder and interaction is a common feature of the fractionalized states. Therefore the fractionalized states are expected to have a dissipationless feature which is a key property on application to quantum information processes [2]. The Majorana fermions (MF) are another example of the fractionalized electron and has been recently discussed in the context of condensed matter physics [3]. Since its field operator in real space satisfies a relation $\gamma = \gamma^\dagger$, MF are often called real fermions and are regarded as *a half* (fraction) of a usual complex fermions.

Superconductors and superfluids are the most promising candidates which host MF because the particle number is not a good quantum number in these systems as required by the MF field. Actually the existence of MF has been discussed in a vortex core or at an edge of the chiral $p + ip$ superconductor [4], ^{3}He and Bose-Einstein condensates [5,6], at an interface between a superconductor and a topological insulator [7], and in a quantum Hall edge (state) with $\nu = 5/2$ [8]. Chiral fermion modes appear only when the time-reversal symmetry $\mathcal{T}$ is broken [1]. Under preserving $\mathcal{T}$ symmetry, the partner with the opposite chirality always coexists. In this case, the edge channels are referred to as *helical*. Along the edge of the two-dimensional quantum spin Hall systems, the helical fermions appear [9]. Analogously, at the edge of the helical superconductors, the helical Majorana fermions appear as the Andreev bound states [10–12]. Noncentrosymmetric superconductors with dominant spin-triplet p-wave order parameter are a realistic playground of helical Majorana fermions [10,11]. In addition, MF excitations are expected in topological superconductors [13]. Thus, understanding of novel phenomena peculiar to MF is highly desired [14]. Although all of the recent developments have assumed noninteracting MF; effects of interaction among MF have been an important open question.

In this Letter, we study theoretically low energy electric transport through the MF modes appearing at edges of two helical superconductors with taking into account the interaction among MF. The model of four interacting MF modes can be mapped into the spinless Tomonaga-Luttinger model by introducing two fictitious chiral fermions. This enables us to analyze low energy physical phenomena of MF by using the bosonization technique. We show that the conductance depends sensitively on the relative helicity of two helical superconductors, the phase difference of two superconductors, and the spin-orbit coupling at the point contact. These are the features of the interference effect unique to Majorana Josephson junctions. Our main results are summarized in Eqs. (9), (12), (14), and (15), and Table I.

Before going into the detailed description, let us explain the physical picture for the results obtained in this Letter. In contrast to the MF in high-energy physics, e.g., neutrinos, those in condensed matter physics can interact with the electromagnetic field even though they are neutral. This is because the phases of the superconducting order parameters enter into the relation between the electron operator and MF operator as given in Eq. (1). In the tunneling Hamiltonian in Eq. (5) in terms of the original electron operators, there are two Hermitian conjugate terms as $\Psi^\dagger_{1\sigma}\Psi_{2\sigma'}$ and $\Psi^\dagger_{2\sigma'}\Psi_{1\sigma}$. These two terms act successively in the tunneling processes, and hence the phase factors of the tunneling matrix element usually cancel out for the quasiparticle tunneling. On the other hand, when Ψ's are expressed by MF operators in Eq. (1), these two terms are combined into a single term with the coefficient depending on the phase difference between the two superconductors (φ) as described in Eqs. (5) and (6). This interference

TABLE I. The temperature dependence of the most dominant terms in conductances given in Eqs. (12) and (15) at low temperature, where φ is the phase difference between the two helical superconductors, $\boldsymbol{\lambda}$ represents spin-orbit interaction at a point contact, and "const" means the conductance independent of temperature. Here $K = 1\,(g = 0)$ represents no interacting case, while $K < 1\,(g > 0)$ and $K > 1\,(g < 0)$ are the interacting cases. The conductance depends on the relative helicity of the two superconductors: the equal helicity configuration (upper column) and the opposite one (lower column)

		$\boldsymbol{\lambda} = 0$	$\boldsymbol{\lambda} \neq 0$
Equal helicity			
$\varphi = 0$	$K = 1$	0	const
	$K < 1$	0	T^{2K-2}
	$K > 1$	0	const
$\varphi \neq 0$	$K = 1$	const	const
	$K < 1$	$T^{2/K-2} \to 0$	T^{2K-2}
	$K > 1$	$T^{2/K-2}$	$T^{2/K-2}$
Opposite helicity			
$\varphi = 0$	$K = 1$	0	const
	$K < 1$	0	const
	$K > 1$	0	$T^{2/K-2}$
$\varphi \neq 0$	$K = 1$	const	const
	$K < 1$	const	T^{2K-2}
	$K > 1$	const	$T^{2/K-2}$

unique to the MF is the basic reason why the quasiparticle tunneling is sensitive to φ. The tunneling process is also sensitive to the relative helicities of the two superconductors. The reason is that the forward tunneling terms remain marginal in the scaling analysis. On the other hand, the backward tunneling terms can be either relevant or irrelevant depending on the sign of interaction. In the equal helicity configuration, the forward tunneling suffers spin flip as shown in Fig. 1. On the other hand in the opposite helicity configuration, the backward tunneling requires spin flip.

We consider the interacting helical edge channels [15] as shown in Fig. 1. By using solutions of the Bogoliubov–de Gennes equation, Majorana fermions at the edge of the helical superconductors are described by [12] $H_0 = iv\sum_{j=1,2}\int dx[\gamma_{Lj}(x)\partial_x\gamma_{Lj}(x) - \gamma_{Rj}(x)\partial_x\gamma_{Rj}(x)]$, where $\gamma_\mu(x)$ for $\mu = (R1, L1, R2, L2)$ are the four species of Majorana fermion field satisfying the anticommutation relation $\{\gamma_\mu(x), \gamma_{\mu'}(x')\} = (1/2)\delta_{\mu,\mu'}\delta(x - x')$. The electron operator is expressed in the low energy sector as

$$\Psi_{j,\sigma}(x) = e^{i\varphi_j/2}e^{i\varphi_\sigma}\chi\gamma_{j,\sigma}(x), \tag{1}$$

$$\chi = \begin{cases} e^{i\pi/4}, & (j,\sigma) = (1,\uparrow) \text{ and } (2,\downarrow) \\ e^{-i\pi/4}, & (j,\sigma) = (1,\downarrow) \text{ and } (2,\uparrow), \end{cases} \tag{2}$$

where φ_j is the phase of superconducting order parameter for the two superconductors indicated by $j = 1$ and 2, $\sigma = \uparrow, \downarrow$ represents pseudospin index, and φ_σ is the relative phase of the pair potential for the two pseudospins. In Eq. (1), we have considered pair potential in two-

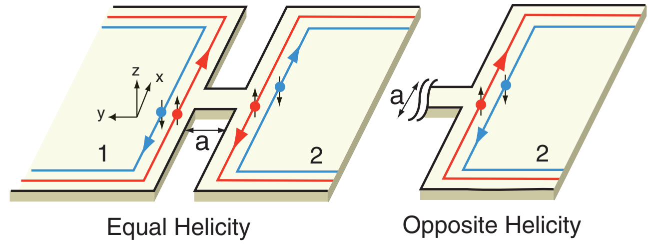

FIG. 1 (color online). Schematic picture of helical Majorana excitations at edges of two helical superconductors.

dimensional ^{3}He-B phase as an example of the helical edge state. As shown in Fig. 1, the pseudospin index $\sigma = \uparrow, \downarrow$ is related to the moving direction of the chiral Majorana fermions (R, L), which are basically determined by the pair potential and the spin-orbit interaction (SOI) in the bulk region.

In addition to H_0, we consider the two terms in the Hamiltonian. At first, the interaction $H_{\text{int.}}$ screened by the bulk states comes from the short-range electron-electron interaction as given by $H_{\text{el-int.}} = \int dx \int dx' C_\alpha^\dagger(x)C_\beta^\dagger(x')V(x - x')C_\beta(x')C_\alpha(x)/2$, where $C_{\alpha(\beta)}$ is the electron operator with α and β labeling the electron spin. The original electron spin α is expressed by the linear combination of pseudospin σ in the presence of the SOI. We take only the possible relevant terms, and neglect the irrelevant interactions including the spatial derivatives. Considering also the fact that $\Psi_{i\sigma}^\dagger\Psi_{i\sigma} \propto (\gamma_{i\sigma})^2 = \text{const.}$, and assuming that the interaction should conserve the electron number in each superconductor, the only remaining interactions derived from $H_{\text{el-int}}$. in low energy are [16]

$$\begin{aligned} H_1 &= U_1 \int dx[\Psi_{1,\uparrow}^\dagger\Psi_{1,\downarrow}\Psi_{2,\uparrow}^\dagger\Psi_{2,\downarrow} + \text{H.c.}] \\ H_2 &= U_2 \int dx[\Psi_{1,\uparrow}^\dagger\Psi_{1,\downarrow}\Psi_{2,\uparrow}\Psi_{2,\downarrow}^\dagger + \text{H.c.}]. \end{aligned} \tag{3}$$

Note here that more than four species of Majorana fermions and SOI are indispensable to having effective interaction [17] and that interaction terms including spatial derivatives are irrelevant in the low-energy limit. Expressing Eqs. (3) by Eqs. (1) and (2), we obtain

$$H_{\text{int.}} = g\int dx\gamma_{R1}(x)\gamma_{R2}(x)\gamma_{L2}(x)\gamma_{L1}(x), \tag{4}$$

with $g = -2U_1\cos(2\varphi_s) + 2U_2$.

The last term is the tunneling Hamiltonian between the two edges represented by

$$\begin{aligned} H_T = -ta\sum_{\sigma,\sigma'}[&\Psi_{1,\sigma}^\dagger(0)\{\sigma_0 + i\boldsymbol{\lambda}\cdot\boldsymbol{\sigma}\}_{\sigma,\sigma'}\Psi_{2,\sigma'}(0) \\ &+ \Psi_{2,\sigma}^\dagger(0)\{\sigma_0 - i\boldsymbol{\lambda}\cdot\boldsymbol{\sigma}\}_{\sigma,\sigma'}\Psi_{1,\sigma'}(0)], \end{aligned} \tag{5}$$

$$\begin{aligned} = 2ita[&\cos(\varphi/2)A_- + \lambda_3\sin(\varphi/2)A_+ \\ &- \cos(\varphi/2)\lambda_-B_+ + \sin(\varphi/2)\lambda_+B_-], \end{aligned} \tag{6}$$

with $A_\pm = \gamma_{1,\uparrow}\gamma_{2,\uparrow} \pm \gamma_{1,\downarrow}\gamma_{2,\downarrow}$, $B_\pm = \gamma_{1,\uparrow}\gamma_{2,\downarrow} \pm \gamma_{1,\downarrow}\gamma_{2,\uparrow}$, $\lambda_- = \lambda_1 \cos\varphi_s - \lambda_2 \sin\varphi_s$ and $\lambda_+ = \lambda_1 \sin\varphi_s + \lambda_2 \cos\varphi_s$, where $\varphi = \varphi_1 - \varphi_2$ is the macroscopic phase difference, $\varphi_s = \varphi_\uparrow - \varphi_\downarrow$ is the difference in the spin-dependent phases, $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ are the Pauli matrices, and σ_0 is the 2×2 unit matrix. The width and the length of a point contact is indicated by a. In Eq. (5), we consider the SOI at the point contact described by $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3)$. By applying electric fields at the point contact, it is possible to induce the Rashba-type SOI as $\boldsymbol{\lambda} = g_{\mathrm{so}}(-E_z, 0, E_x)$, where g_{so} is a coupling constant, E_x and E_z correspond to the potential gradient in the x and z direction, respectively. When the Dresselhous-type SOI can be introduced, λ_2 also becomes nonzero value. The operator of the electric current is calculated from the equation $J = e\partial_t \sum_\sigma \int dx \Psi^\dagger_{1,\sigma}(x)\Psi_{1,\sigma}(x)$. We find that a relation $J(\varphi) = eH_T(\varphi + \pi)$ always holds. We assume that the pseudospin of the right(left)-mover is $\uparrow(\downarrow)$ at the edge 1 as shown in Fig. 1. Therefore we choose $\gamma_{1,\uparrow} = \gamma_{R1}$ and $\gamma_{1,\downarrow} = \gamma_{L1}$. At the edge 2, we choose $\gamma_{2,\uparrow} = \gamma_{L2}$ and $\gamma_{2,\downarrow} = \gamma_{R2}$ for the equal helicity configuration, and $\gamma_{2,\uparrow} = \gamma_{R2}$ and $\gamma_{2,\downarrow} = \gamma_{L2}$ for the opposite helicity configuration.

To analyze the Hamiltonian, we introduce a complex fermion field by $\psi_A(x) = \gamma_{A1}(x) + i\gamma_{A2}(x)$ and $\psi^\dagger_A(x) = \gamma_{A1}(x) - i\gamma_{A2}(x)$ with $A = R$ and L. These operators satisfy the usual fermion anticommutation relations: $\{\psi_A(x), \psi_{A'}(x')\} = 0$ and $\{\psi_A(x), \psi^\dagger_{A'}(x')\} = \delta_{A,A'}\delta(x - x')$. We rewrite the Hamiltonian $H_0 + H_{\mathrm{int.}}$ in terms of these complex fermion operators as

$$H_0 + H_{\mathrm{int.}} = -iv \int dx[\psi^\dagger_R(x)\partial_x\psi_R(x) - \psi^\dagger_L(x)\partial_x\psi_L(x) + \frac{g}{4}\psi^\dagger_R(x)\psi_R(x)\psi^\dagger_L(x)\psi_L(x)] + \mathrm{const.} \tag{7}$$

This Hamiltonian is exactly that of the massless Tomonaga-Luttinger model. It is extremely simple as compared to that of interacting helical edge fermion systems [18]. In the standard bosonization method, $(\partial_x/2\pi)\phi_{L(R)}(x) =: \psi^\dagger_{L(R)}(x)\psi_{L(R)}(x)$, $\phi(x) = \phi_R(x) + \phi_L(x)$, and $\theta(x) = \phi_R(x) - \phi_L(x)$, Eq. (7) is transformed into

$$H_0 + H_{\mathrm{int.}} = \frac{\tilde{v}}{8\pi}\int dx \frac{\{\partial_x\phi(x)\}^2}{K} + K\{\partial_x\theta(x)\}^2, \tag{8}$$

where $\tilde{v} = v\sqrt{1 - \tilde{g}^2}$ and $K = \sqrt{(1 - \tilde{g})/(1 + \tilde{g})}$ with $\tilde{g} = g/(8\pi v)$ [19,20].

We first discuss the tunneling effect in the *equal helicity configuration*. By using the bosonization technique, the tunneling Hamiltonian becomes

$$H_T = \frac{ta}{\pi}\left[\frac{\lambda_+}{2}\sin\left[\frac{\varphi}{2}\right]\partial_x\theta(x) - \frac{\lambda_-}{2}\cos\left[\frac{\varphi}{2}\right]\partial_x\phi(x)\right]_{x=0} + \frac{i\eta_R\eta_L ta}{\pi\alpha_0}\left[\cos\left[\frac{\varphi}{2}\right]\sin\theta(0) - \lambda_3\sin\left[\frac{\varphi}{2}\right]\sin\phi(0)\right],$$

where η_R and η_L are the Klein factor. In the above equation, the terms including $\partial_x\theta(x)$ and $\partial_x\phi(x)$ represent the forward tunneling process: hopping of the left(right) mover to the left(right) mover. On the other hand, the terms including $\sin\theta(0)$ and $\sin\phi(0)$ represent the backward tunneling: hopping of the left(right)-mover to the right(left)-mover. Before turning into the conductance, the Josephson current should be clarified. Within the second-order perturbation expansion, we find

$$J = e\Delta\left[\frac{at}{\pi v}\right]^2 \sin\varphi\left[1 - \lambda_3^2 - \frac{1}{K}\lambda_+^2 + K\lambda_-^2\right], \tag{9}$$

where we have used $(\eta_R\eta_L)^2 = -1$, $(\alpha_0)^{-1} = k_F \equiv \Delta/v$, and Δ is the amplitude of pair potential at sufficiently low temperature $T \ll T_c$ with T_c being superconducting transition temperature. The ground state of junction is at $\varphi = 0$ for $\boldsymbol{\lambda} = 0$ [21]. Equation (9) does not recover a usual relation $J \propto (1 + \boldsymbol{\lambda}^2)$ expected in s-wave Josephson junction even in the absence of interaction, (i.e., $K = 1$). This is a characteristic feature of Majorana fermion excitation.

On the basis of the linear response theory, we calculate DC conductance of the point contact using the standard Kubo formula, $\sigma = -\lim_{\omega\to 0^+}[Q^R(\omega) - Q^R(0)]/(i\omega)$, where the correlation function is obtained by $Q^R(\omega) = Q(i\omega_n \to \omega + i\delta)$ with $Q(\omega_n) = -\int_0^{1/T} d\tau e^{i\omega_n\tau}\langle J(\tau)J(0)\rangle$, τ is the imaginary time, and ω_n is the Matsubara frequency. The following four terms contribute to $Q(\omega_n)$,

$$\langle J(\tau)J(0)\rangle = \langle F_\theta^2\partial_x\theta(\tau)\partial_x\theta(0) + F_\phi^2\partial_x\phi(\tau)\partial_x\phi(0)\rangle_{x=0} + \langle B_\theta^2\sin\theta(\tau)\sin\theta(0) + B_\phi^2\sin\phi(\tau)\sin\phi(0)\rangle_{x=0},$$

where $F_\theta = \cos(\varphi/2)\lambda_+/2$, $F_\phi = \sin(\varphi/2)\lambda_-/2$, $B_\theta = \sin(\varphi/2)/\alpha_0$ and $B_\phi = \cos(\varphi/2)\lambda_3/\alpha_0$. From the scaling analysis [22–24], it is concluded that B_θ and B_ϕ are relevant for $K > 1$ $(g < 0)$ and $K < 1$ $(g > 0)$, respectively.

The forward tunneling terms are calculated to be

$$\langle \partial_x\mathcal{B}(x,\tau)\partial_x\mathcal{B}(x,0)|_{x=x_0}\rangle = X(\tau), \tag{10}$$

$$X(\tau) = \frac{8k_F}{v}\delta(\tau) - \frac{4\pi}{v^2}T\sum_{\omega_n} e^{-i\omega_n\tau}|\omega_n|. \tag{11}$$

where $\mathcal{B} = \sqrt{K}\theta$ or $\phi/\sqrt{K}$. Finally, we reach the dc conductance for the *equal helicity configuration*

$$\frac{\sigma}{G_0} = \pi\frac{\lambda_+^2}{K}\cos^2\left(\frac{\varphi}{2}\right) + \sin^2\left(\frac{\varphi}{2}\right)D_\theta\left(\frac{T}{T_0}\right)^{2/K-2} + \pi\lambda_-^2 K\sin^2\left(\frac{\varphi}{2}\right) + \lambda_3^2\cos^2\left(\frac{\varphi}{2}\right)D_\phi\left(\frac{T}{T_0}\right)^{2K-2}, \tag{12}$$

where $G_0 = (tae/\pi v)^2$ and $D_{\mathcal{A}}(T_0) = \Delta_0^2\int_0^{1/T_0} d\tau\tau\langle\sin\mathcal{A}(\tau)\sin\mathcal{A}(0)\rangle$ for $\mathcal{A} = \theta$ and ϕ are the correlation function at $T = T_0 < T_c$. Remarkably, the conductance depends on the phase difference between the two superconductors. This stems from a peculiar feature of

Majorana fermion. At $\varphi = 0$, the conductance vanishes in the absence of the SOI, (i.e., $\boldsymbol{\lambda} = \mathbf{0}$). In the presence of the SOI at $\varphi = 0$, the last term is relevant for $K < 1$ in addition to the first term. The first term is independent of T, whereas the last term increases with decreasing T as T^{2K-2} for $K < 1$. For $\varphi \neq 0$, the second term is relevant for $K > 1$ even in the absence of the SOI. Finally for $\varphi \neq 0$ and $\boldsymbol{\lambda} \neq \mathbf{0}$, all terms contribute to the conductance. The argument above is summarized in Table I.

The total current through a Josephson junction is described by the so-called resistively and capacitively shunted junction model

$$J = \frac{C}{2e}\frac{d^2\varphi}{dt^2} + \frac{1}{2eR(\varphi)}\frac{d\varphi}{dt} + J_0 \sin(\varphi), \tag{13}$$

with C being the capacitance of a junction. In the present junction, the resistance $R = 1/\sigma$ depends on φ. Thus Majorana fermion excitation would modify dynamics of a Josephson junction. The phase φ may be determined self-consistently so that the current can be optimized. Such an issue is a natural extension of this Letter.

We show the Josephson current and the conductance in the *opposite helicity configuration* as follows:

$$\langle J \rangle = e\Delta\left[\frac{at}{\pi v}\right]^2 \sin\varphi\left[\frac{1}{K} - \lambda_3^2 K - \lambda_+^2 + \lambda_-^2\right], \tag{14}$$

$$\frac{\sigma}{G_0} = \pi\frac{\sin^2(\varphi/2)}{K} + \lambda_+^2\cos^2\left(\frac{\varphi}{2}\right)D_\theta\left(\frac{T}{T_0}\right)^{2/K-2} + \pi\lambda_3 K\cos^2(\varphi/2) + \lambda_-^2\sin^2\left(\frac{\varphi}{2}\right)D_\phi\left(\frac{T}{T_0}\right)^{2K-2}. \tag{15}$$

For $\boldsymbol{\lambda} = \mathbf{0}$, the conductance proportional to $\sin^2(\varphi/2)$ is independent of temperature, which is in sharp contrast to that in the equal helicity case given in Eq. (12). The behaviors of the conductance σ are summarized in the Table I.

In summary, we have studied electric transport through a point contact which connects Majorana fermion modes appearing at the edges of two helical superconductors by taking into account the interaction among Majorana fermions and the spin-orbit interaction (SOI) at a point contact. By introducing a fictitious fermion consisting of two Majorana fermions, the Majorana fermion model is transformed into the Tomonaga-Luttinger model. The application of the standard bosonization technique enables to analyze low energy physical phenomena of a Majorana fermion. It is found that several novel features appear due to the Majorana fermions such as (i) the conductance is sensitive to the phase difference of two superconductors, (ii) tunneling with SOI gives quite distinct behavior from that without SOI, (iii) the transport phenomena depend on relative helicity of two superconductors as shown in Eqs. (12) and (15), and (iv) the interactions leads to the power-law temperature or voltage dependences.

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