Josephson interferometer in a ring topology as a proof of the symmetry of $\text{Sr}_2\text{RuO}_4$  

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The Josephson effect is theoretically studied in two types of SQUIDs consisting of $s$ wave superconductor and $\text{Sr}_2\text{RuO}_4$. Results show various response of the critical Josephson current to applied magnetic fields depending on the type of SQUID and on the pairing symmetries. In the case of a $p, +ip$, wave symmetry, the critical current in a corner SQUID becomes an asymmetric function of magnetic fields near the critical temperatures. Our results well explain a recent experimental finding [Nelson et al., Science 306, 1151 (2004)]. We also discuss effects of chiral domains on the critical current.  

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I. INTRODUCTION  

One of the important developments for unconventional superconductivity after the discovery of high-$T_c$ superconductivity, has been the series of so-called phase sensitive experiments which test for the symmetry of the Cooper pair wave function. These are the SQUID-type of interference experiments, the observation of spontaneous half-flux quantization in frustrated loops and the measurement of zero-bias peaks in quasiparticle tunneling spectra indicating subgap quasiparticle states at the sample surface. This set of experiments, technically rather diverse, is based on the same concept, the unconventional phase structure of the superconducting condensate, and have uniquely proven that the Cooper pairs have the spin-singlet $d_{x^2-y^2}$ wave symmetry. A further unconventional superconductor whose pairing symmetry has been established with high confidence is $\text{Sr}_2\text{RuO}_4$. This is a spin triplet $p$-wave superconductor with a gap function of the form $d(k) \propto \hat{z}(p_x \pm ip_y)$, a so-called chiral $p$-wave state with whose Cooper pairs possess an angular momentum component along the $z$ axis. Tunneling experiments have shown the presence of subgap surface states. Moreover, the anomalous temperature dependence of the critical current in $\text{Pb}/\text{Sr}_2\text{RuO}_4/\text{Pb}$ Josephson junction arrangement has been interpreted in terms of an interference effect. A direct experiment of the type of a SQUID-interference or frustrated loop is difficult here for many technical reasons and has not been performed until very recently.  

In this paper we would like to analyze some issues which have to be taken into account in the interpretation of the SQUID-type experiments for $\text{Sr}_2\text{RuO}_4$. The basic principle had been designed long ago by Geshkenbein and co-workers. One of the problems lies in the Josephson junctions between a conventional spin-singlet $s$-wave superconductor and a spin-triplet $p$-wave superconductor. The mismatch of the angular moment (parity) and of spin quantum number on the two sides of a Josephson junction seems at first sight inhibit the lowest order Josephson effect so that only a coupling in second order would be allowed. This would indeed be fatal for phase sensitive tests based on the Josephson effect. It has, however, been shown that the presence of spin-orbit coupling saves the situation since only the total angular momentum has to be conserved. Thus under well-defined conditions, the lowest order between an $s$- and a $p$-wave superconductor is possible. The conditions leave a certain arbitrariness concerning the sign of the Josephson coupling which can be important for interference effects. This can be illustrated, if we consider the definition of the lowest order matrix element which can be derived from a simple microscopic tunneling model:  

$$\langle \psi(k)(k \times n) \cdot d(k) \rangle_{FS},$$  

where the average runs over the Fermi surface and $n$ is the interface normal vector. The arbitrariness appears in the orientation of the normal vector, into or out of the $p$-wave superconductor. This has been recently demonstrated by Asano and co-workers for a model where the spin-orbit coupling of the interface potential was taken into account for this matrix element whose sign then depends on the shape of the interface potential. Thus details of the spin-orbit coupling and the interface potential, e.g., the way the parity is broken at the interface, matter for the Josephson phase relation. The arrangement suggested by Geshkenbein et al. relies on the assumption that all interfaces treat parity the same way. We will follow this assumption here too.  

We discuss now two basic forms of SQUID-interference devices built from an $s$- and a $p$-wave superconductor. The first type in Fig. 1(a) is the symmetric SQUID as proposed...
by Geshkenbein and co-workers\textsuperscript{25} and the second in Fig. 1(b) is the corner-SQUID analogous to the one used for high-$T_c$ superconductors. The $p_x+ip_y$ phase of Sr$_2$RuO$_4$ introduces an additional problem. This phase is twofold degenerate, i.e., $p_x+ip_y$ and $p_x-ip_y$ are equivalent and would in general form domains in a sample, depending on the cooling history. We will discuss in the following also the implications domains in a sample, depending on the cooling history. We will discuss the critical current on the interference experiment. Similar problems appear for other chiral superconducting phase as we will discuss below.

This paper is organized as follows. In Sec. II, the Josephson effect in the corner SQUID of $p_y$ symmetry is identical to that of the $p_z$ symmetry.

II. JOSEPHSON CURRENT

Before discussing the SQUID-experiment, the basics of the Josephson current-phase relation between $s$ wave superconductor and Sr$_2$RuO$_4$ (SRO) in Fig. 1(c) has to be addressed. We describe the gap function of Sr$_2$RuO$_4$ by\textsuperscript{16,31}

$$\Delta_p = i(\eta_\parallel \vec{p}_x + \eta_\perp \vec{p}_y) \hat{\vec{z}} \cdot \hat{\vec{\sigma}}_2, \quad (2)$$

$$= i\Delta(\vec{p}_x \pm i\vec{p}_y) \hat{\vec{z}} \cdot \hat{\vec{\sigma}}_2 \quad (3)$$

$$= i\Delta e^{\nu \hat{\vec{z}} \cdot \hat{\vec{\sigma}}_2}, \quad (4)$$

where $\hat{\vec{\sigma}}_j$ with $j=1, 2$ and 3 are the Pauli matrices, $\eta_\parallel$ and $\eta_\perp$ are the two complex order parameters and $\hat{\vec{z}}$ is taken as a unit vector parallel to the $z$ axis.\textsuperscript{17} Moreover, $\vec{p}_x = p_x/p_F = \cos \theta \vec{p}_x = p_x/|p_F| = \sin \theta$ is the normalized momentum component on the Fermi surface in the $x(y)$ direction with $p_F$ being the Fermi momentum of the $p$-wave superconductor. Assuming a cylindrical symmetric Fermi surface, we can represent this gap function also simply to the angle $\theta$ on the Fermi surface, reflecting best its internal phase structure. The gap function of the $s$-wave superconductor is given by $\Delta_k = i\Delta \delta_z$. Without any loss of generality, we may take the gap magnitudes identical in both superconductors, $\Delta$. On the basis of the current-phase relation,\textsuperscript{27–29} the Josephson current for the lowest two orders derived from a microscopic calculation close to $T_c$ can be written as

$$\bar{J}_{p_xip_y} = J_1 \cos(\varphi + \theta_n) - J_2 \sin(2(\varphi + \theta_n)). \quad (5)$$

$$J_1 = T_B \alpha_s \left( \frac{\Delta}{2T} \right)^\frac{3}{3}, \quad (6)$$

$$J_2 = T_B \beta \left( \frac{\Delta}{2T} \right)^\frac{3}{3}, \quad (7)$$

where $\theta_n$ is the angle of the junction normal vector in the plane relative to the $x$ axis, $T_B$ denotes the transmission probability of a Cooper pair and $\alpha_s$ is a measure for the strength of the spin orbit coupling. Note that it can have either sign depending on the junction. The Josephson current $\bar{J}$ is measured in units of $e\Delta_0N_c/h$, where $\Delta_0$ is the amplitude of the gap function at $T=0$ and $N_c$ is the number of propagating channels on the Fermi surface. Constant coefficients of the order of unity have been omitted in Eq. (5). From Eq. (1) it is clear that the coupling between $s$-wave and $p_x+ip_y$-wave superconductor for a junction in $x$ direction ($\theta_n=0$) goes via the $p_x$ component. We assume a gauge where the overall phase of the $p$-wave order parameter is shifted by $\pm \pi/2$ for the $p_y$ component. Thus, in lowest order the current-phase relation is proportional to $\cos \varphi$ rather than $\sin \varphi$, in Eq. (5).

In the first step, we consider the interference pattern of the two basic SQUID’s in Figs. 1(a) and 1(b). We assume that all junctions are of the same in the sense, that the normal vector in Eq. (1) is pointing a definite direction, say towards the $s$-wave superconductor. In the symmetric SQUID, the Josephson current of the left and right junction are given by

$$J_L(\varphi) = J_1 \cos \varphi - J_2 \sin 2\varphi, \quad (8)$$

$$J_R(\varphi) = -J_1 \cos \varphi - J_2 \sin 2\varphi. \quad (9)$$

The relative sign change between the first-order terms on both sides is due to the corresponding $\theta_n$ which differ by $\pi$. The Josephson current in the symmetric SQUID is then expressed by\textsuperscript{25}

$$J_S(\varphi, \Phi) = J_L(\varphi + \phi_B) + J_R(\varphi - \phi_B), \quad (10)$$

$$= -2J_1 \sin \varphi \sin \phi_B - 2J_2 \sin 2\varphi \cos 2\phi_B, \quad (11)$$

where $\phi_B = \pi\Phi/\Phi_0$ and $\Phi_0 = 2\pi\hbar c/e$. In the corner SQUID,
the Josephson current in the bottom junctions is given by

\[ J_b(\phi) = -J_1 \sin \phi + J_2 \sin 2\phi, \]  

(12)

where the current-phase relation is derived from Eq. (5) with \( \theta_0 = \pi/2 \). The Josephson current in the corner SQUID is expressed in the same way,

\[ J_c(\phi, \Phi) = J_1 \cos(\phi + \phi_B) - J_1 \sin(\phi - \phi_B) - 2J_2 \cos 2\phi \sin 2\phi_B. \]  

(13)

Note that for the corner SQUID it matters which state is realized. For Eq. (12) we assumed that the state \( p_x + ip_y \), while for \( p_x - ip_y \) would yield a sign change of the first term.

In Fig. 2, the critical Josephson current in the symmetric SQUID and in the corner SQUID are shown as a function of \( \Phi \). A parameter \( J_1/J_2 = 10 \) is realized in high temperatures near \( T_c \). Since \( J_1 \gg J_2 \), we find

\[ \text{max}\left| J_b(\phi) \right| = 2J_1 \left| \sin \left( \frac{\pi}{\Phi_0} \phi \right) \right|. \]  

(14)

\[ \text{max}\left| J_c(\phi, \Phi) \right| = 2J_1 \left| \sin \left( \frac{\Phi}{\Phi_0} \pm \frac{\pi}{4} \right) \right|. \]  

(15)

The odd parity symmetry immediately results in a minimum of the critical current at \( \Phi = 0 \) in the symmetric SQUID, which is connected with the opposite sign of Josephson coupling on the two junctions according to Eq. (9). Note that this pattern is not dependent on which of the two degenerate \( p \)-wave states is realized. Actually for the symmetric SQUID any \( p \)-wave pairing state gives the same qualitative behavior and does not depend on the detailed symmetry, such as broken time reversal symmetry, as long as the first-order Josephson coupling induced by spin-orbit effects is dominant. This is different for the corner SQUID which leads to a distinction between different \( p \)-wave states.

The change to the corner SQUID configuration yields a phase shift by \( \pm \pi/2 \) for \( p_x \pm ip_y \). As a consequence, the \( \cos \phi \) current-phase relation can be exchanged by \( \sin \phi \) for the bottom junction. For comparison, the results for the \( p_x \) and \( p_y \)-wave states are depicted in Fig. 2 with the corresponding device illustrations in Figs. 1(d)–1(f). The Josephson current-phase relation in the single junctions can for fixed gauge be given by \( 30,33 \)

\[ J_{p_x}(\phi) = -J_2 \sin 2\phi. \]  

(16)

\[ J_{p_y}(\phi) = J_1 \sin \phi - J_2 \sin 2\phi. \]  

(17)

For the \( p_x \) component (\( n \) parallel to \( x \)), there is no contribution due to spin-orbit coupling, in contrast to the \( p_y \) component which has such a term proportional to \( \sin \phi \) in the \( p_y \) symmetry.

In the symmetric SQUID of the \( p_x \) type, the critical current has the period of \( \Phi_0/2 \) as shown in Fig. 2(a) because there is only the second-order contribution in the Josephson coupling. On the other hand, the \( p_y \) type behaves identical to the \( p_x + ip_y \) type. For the corner SQUID configuration the \( p_x \) symmetry (and equivalently for the \( p_y \) symmetry) has again a \( \Phi_0/2 \)-periodic interference pattern of the critical current.

If we assume that \( J_1 \ll J_2 \) due to weak spin-orbit coupling, the features of the interference pattern are significantly modified. In Fig. 3, the critical Josephson current is plotted as a function of \( \Phi \) for \( J_1 = 0.1J_2 \). For \( J_1 \ll J_2 \), we actually find

\[ \text{max}\left| J_b(\phi) \right| = 2J_1 \cos \left( 2\pi \frac{\phi}{\Phi_0} \right), \]  

(18)

\[ \text{max}\left| J_c(\phi, \Phi) \right| = 2J_1 \sin \left( 2\pi \frac{\phi}{\Phi_0} \right), \]  

(19)

for the \( p_x + ip_y \) symmetry. The period of \( J_c \) is given by \( \Phi_0/2 \) because the Josephson current proportional to \( \sin 2\phi \) is dominant. The critical current in the symmetric SQUID takes its maxima at \( \Phi = 0 \) because \( \sin 2\phi \) remains unchanged under the \( \pi \) phase shift. On the other hand in the corner SQUID, the critical current takes its minima at \( \Phi = 0 \). This is due to the sign change of \( \sin 2\phi \) under the \( \pi/2 \) phase shift in the \( p_x + ip_y \) symmetry. The critical current in the \( p_x \) and \( p_y \) symmetries can be described by Eq. (18) irrespective of types of SQUID as shown in the Fig. 3. Thus the minima at \( \Phi = 0 \) in the corner SQUID directly suggests the \( p_x + ip_y \) symmetry in SRO when the period of the oscillations is \( \Phi_0/2 \).

In Fig. 4, the critical current for the \( p_x + ip_y \) symmetry is shown for several choices of \( J_1/J_2 \). In the vertical axis, successive plots have been offset. The critical current is always symmetric with respect to \( \Phi \) in the symmetric SQUID. The current maxima at \( \Phi_0 \) for \( J_1/J_2 = 0.1 \) is changed to the
to the analysis of the above SQUID configurations which we will discuss here for a few simple situations as shown in Fig. 5, under the assumption $J_1 \gg J_2$.

The presence of horizontal domain walls in Fig. 5, i.e., $n \parallel y$, implies that the two chiral domains have either $p_x + ip_y$ or $-p_x + ip_y$ form. According to our previous discussion, no qualitative change of the interference pattern is expected. Thus the critical current in Fig. 2(a) remains unchanged even in the presence of the horizontal domain walls in the symmetric SQUID. On the other hand, the vertical domain walls give rise to domains with $p_x + ip_y$ and $p_x - ip_y$ form. In this case the number for domain walls between the two junctions matters since each domain wall switches the phase of $p_y$, entering the first-order term of the Josephson coupling, by $\pi$. Thus the interference pattern would look different for an odd or even number of domain walls:

$$
\max |J_S(\Phi)| = \begin{cases} 
2J_1 \sin (\pi \frac{F_1}{F_0}) N_{DW} \text{ even}, \\
2J_1 \cos (\pi \frac{F_1}{F_0}) N_{DW} \text{ odd},
\end{cases}
$$

where $N_{DW}$ is the number of the vertical domain walls. If the number of walls is an even integer, the critical current takes its minimum at $\Phi = 0$ as shown in Fig. 6(a). However, the critical current has its maximum at $\Phi = 0$ when the number of domain walls is an odd integer. Thus the existence of the vertical domain walls can change drastically the characteristic feature of the critical current in the symmetric SQUID.

In the corner SQUID, the current-phase relation in the two junctions as shown in Fig. 5(b) are given by

$$
J_A(\varphi) = J_1 \cos \varphi - J_2 \sin 2\varphi, 
$$

$$
J_B(\varphi) = \pm J_1 \sin \varphi + J_2 \sin 2\varphi,
$$

where the sign of the first-order term of the bottom junction is determined by the chirality of the last domain at the bottom. The number of domain walls is irrelevant here. Still the asymmetry of the interference pattern indicates the broken time reversal symmetry in any case as shown in Fig. 6(b).

### IV. OTHER CHIRAL SUPERCONDUCTING PHASES

The chirality of superconductivity reflects the internal angular momenta of a Cooper pair. Also other superconducting phases besides the chiral $p$-wave phase can be found with this property. In this section, we compare the Josephson effect of the $p_x + ip_y$-wave phase with those of other chiral phase of $d$- and $f$-wave origin.

A chiral phase in the case of spin-singlet $d$-wave symmetry can be composed of the $d_{x^2-y^2}$ and the $d_{xy}$ state, when they are degenerate, yielding a $d_{x^2-y^2} \pm id_{xy}$. This gap function was discussed as the surface states of high-$T_c$ cuprates where the degeneracy is not given, but one of the components is subdominant. The other example is Na$_2$CoO$_2$ · yH$_2$O. Thus we consider the gap function
\[ \Delta^{(0)}_k = \{ \eta_1 (p_x^2 - p_y^2) + \eta_2 \bar{p}_x \bar{p}_y \} i \hat{\sigma}_2 \]
\[ = \Delta (p_x^2 - p_y^2 \pm i 2 \bar{p}_x \bar{p}_y) i \hat{\sigma}_2 = \Delta e^{\pm i 2 \theta} i \hat{\sigma}_2. \]

Here the coupling to the \( s \)-wave superconductor is simpler, since it is not relying on spin-orbit coupling. Thus the current phase relation is dominated by the first-order coupling,

\[ J(\phi) = J_0 \sin(\phi + 2 \theta_0), \]
\[ J_0 = T_B \left( \frac{\Delta}{2 F} \right) \left( \frac{\Delta}{\Delta_0} \right). \]

Concerning the domain wall structure of this state, the two order parameter components behave in the same way as in the \( p \)-wave case. Thus only the component perpendicular to the domain wall normal vector changes sign. Thus, for \( n \parallel x \) the component \( \eta_1 (d_{2+y}) \) changes sign while \( d_{2-z^2} \) keeps the phase. Thus, irrespective of the domain wall number for the vertical domain wall case, the maximum of the critical current in the symmetric SQUID would be at \( \Phi = 0 \). Thus, domain walls could not lead to an interference pattern like in the \( p \)-wave case. In the case of horizontal domain walls, the situation is more complicated. There is a staggered phase for the \( d_{2-z^2} \) component. This would give rise to compensating contributions of the two domains to the Josephson effect, similar to the situation discussed in the context of \( c \)-axis junction between an \( s \)-wave and a twinned high-temperature superconductor.\(^{37,38}\) In addition, this configuration could give rise to spontaneous half-quanta flux lines wherever a domain wall hits the junction perpendicularly. Under these circumstances, the interpretation of interference pattern would require much more care. Note that such spontaneous fluxes do not occur in the \( p \)-wave case as long as the domain wall ends perpendicularly on the junction. In both cases, the flux magnitude depends on the angle between junction interface and domain wall. A similar problem with the domain walls occurs for the corner junction as a simple examination reveals.

The spin-triplet chiral \( f \)-wave symmetry is also a possible candidate for the superconducting phase in \( \text{Na}_x \text{CoO}_2 \cdot \gamma \text{H}_2 \text{O} \). The proposed gap function is given by

\[ \Delta^{(f)}_k = i \Delta \left( p_x^2 - 3 p_y^2 \right) \hat{p}_x \pm i (p_y^2 - 3 p_x^2) \hat{p}_y \cdot \hat{\sigma}_2 \]
\[ = \Delta e^{\pm i 3 \theta} \hat{z} \cdot \hat{\sigma}_2. \]

The current phase relation for the chiral \( f \) wave symmetry is given by Eq. (5). In both the symmetric and the corner SQUID, the characteristic behavior of the critical current for the chiral \( p \) wave symmetry discussed in Secs. II and III are also valid for the chiral \( f \) wave symmetry. In the limit of \( J_1 / J_2 \gg 1 \), the critical current in the symmetric SQUID take the minima at \( \Phi = 0 \) and that in the corner SQUID shows the phase shift by \( \pi / 4 \) in the absence of the domain walls. In addition, there are no differences in effects of domain walls on the critical current between the chiral \( p \) wave and the chiral \( f \) one. To distinguish the chiral \( p \) wave symmetry from the chiral \( f \) wave, we should design SQUID’s with \( \alpha = \pi / 3 \) or \( 2 \pi / 3 \).\(^{39}\) The critical current for \( f \) waves becomes symmetric function of \( \Phi \) and takes its minimum (maximum) at \( \Phi = 0 \) in the SQUID with \( \alpha = \pi / 3 (\alpha = 2 \pi / 3) \).\(^{39}\) As shown in Fig. 7. On the other hand, the critical current for \( J_1 / J_2 \ll 1 \) in the chiral \( p \) wave results in

\[ \max |J_{\alpha = \pi / 3}(\Phi)| = 2 J_1 \left| \cos \left( \frac{\Phi}{\Phi_0} - \frac{\pi}{6} \right) \right|, \]
\[ \max |J_{\alpha = 2 \pi / 3}(\Phi)| = 2 J_1 \left| \cos \left( \frac{\Phi}{\Phi_0} - \frac{\pi}{3} \right) \right|. \]

The calculated results are shown in Fig. 7. The asymmetry in the critical current persists in the chiral \( p \) wave in such SQUID’s. This argument, however, is valid when the differ-

FIG. 6. The critical current in the presence of domain walls for \( J_1 / J_2 = 10 \). In the symmetric SQUID, the results for the chiral \( p \) wave symmetry in the presence of the vertical domain walls are shown in (a). In (b), the critical current for the chiral \( p \) wave symmetry in the corner SQUID is shown. The solid (broken) line denote the results when the chirality of the last domain at the bottom is \( p_x + i p_y \) (\( p_x - i p_y \)) in (b).

FIG. 7. The critical current for the chiral \( p \) wave is compared with that for the chiral \( f \) wave in (a) and (b), where \( J_1 / J_2 = 10 \). The relative junction angle is set at \( \alpha = \pi / 3 \) in (a) and \( \alpha = 2 \pi / 3 \) in (b) as shown in the upper figures.
ence in the transmission probabilities of the two tunnel junctions in Fig. 7 are much smaller than themselves.

V. DISCUSSION

We have shown that the asymmetry of the critical current with respect to $\Phi$ in the corner SQUID is the evidence of the $p_x + ip_y$ symmetry when the period of oscillations is $\Phi_0$. A recent experiment shows a large asymmetry of the critical current in the corner SQUID. The experiment also shows a minima of the critical current around $\Phi = 0$ in the symmetric SQUID. The position of minima, however, slightly deviates from $\Phi = 0$. We have assumed that $J_1$ in the left junction is equal to that in the right junction. When those are different from each other, the Josephson critical current becomes asymmetric of $\Phi$ even in the symmetric SQUID. Thus the small asymmetry found in the experiment might be caused by the asymmetry of the two tunnel junctions. The degree of asymmetry is negligible when the difference of $J_1$ in the two junctions is much smaller than themselves.

In addition to $p_x + ip_y$, the sin($p_x$) + sin($p_y$) and sin($p_x + p_y$) + i sin($p_x - p_y$) symmetries have been proposed so far. The perturbation expansion also indicates a possibility of the chiral $p$ wave symmetry with more complicated structure than sin($p_x$) + i sin($p_y$) and sin($p_x + p_y$) + i sin($p_x - p_y$). The characteristic behavior of the Josephson current in the two types of SQUID discussed in Secs. II and III are valid not only for the isotropic $p_x + ip_y$ symmetry but also for anisotropic sin($p_x$) + i sin($p_y$) and sin($p_x + p_y$) + i sin($p_x - p_y$) symmetries. The latter two gap function have the fourfold symmetry in the momentum space consistently with a thermal conductivity experiment. In particular, sin($p_x + p_y$) + i sin($p_y - p_x$) is close to the chiral $f$ wave symmetry because sin($p_x + p_y$) and sin($p_y - p_x$) change their sign six times on the Fermi surface. To distinguish one symmetry from another, much more detail analysis would be required. At present, we propose the angle resolved tunneling spectra for this analysis.

VI. CONCLUSION

We have studied the critical Josephson current in two types of SQUID consisting of $s$ wave superconductor and Sr$_2$RuO$_4$. In the $p_x + ip_y$, symmetry, the critical Josephson current in the corner SQUID becomes the asymmetric function of $\Phi$ because the current-phase relation in the two junctions relate to each other by $\pi/2$ phase shift in the gap function. We also show that the asymmetry remains even in the presence of the chiral domain structures in Sr$_2$RuO$_4$. Our results well explain the recent experimental findings.

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We do not consider spatial dependence of the pair amplitude. Although the amplitude at the interface deviates from its bulk value, effects of spatial dependence in the pair amplitude are negligible in low energy transport such as the Josephson current. See Y. S. Barash, A. M. Bobkov, and M. Fogelström, Phys. Rev. B 64, 214503 (2001).

Here we assume temperatures are still higher than $\Delta_0/T_B$. When $T \ll \Delta_0/T_B$, the zero-energy state causes the anomalous Josephson effect in the $p_x$ symmetry (see Ref. 30).